SEMANTIC RELATIVITY: A GEOMETRIC FIELD THEORY FOR OBSERVER-DEPENDENT MEANING

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ABSTRACT

In systems where observers are structurally coupled to the fields they measure, semantic geometry and its evolution cannot be specified independently of one another. Seven axioms and a four-term Lagrangian balancing coherence flow, attractor binding, autopoietic generation, and regulatory constraint on a differentiable semantic manifold together constrain three coupled field equations that close a causal loop. A coherence field flows on a dynamic metric; semantic mass, the product of reflexive density, constraint density, and attractor stability, sources curvature through coupling structurally analogous to general relativity; and the resulting curvature reshapes the conditions under which subsequent coherence flows. Noether symmetries yield three conservation laws, including a coherence-sector Hamiltonian whose individual conservation breaks down during rapid metric evolution, paralleling the gravitational energy problem arising in GR. Five necessary and jointly sufficient conditions whose necessity follows from the field equations—self-model, recursive closure, coherence stability, autopoietic self-maintenance, and intrinsic regulation—specify a manifold region constituting a self-directing agent. Twelve orthogonal signatures partition characteristic departures from agency maintenance into rigidity, fragmentation, inflation, and distortion classes, each undermining one or more of the geometric preconditions sustaining the agency criteria. Because each observer's metric is independently constructed, semantic measurement is frame-dependent; the geometric conditions under which independent observers develop compatible metrics through iterated coupling are identified. Four structural falsifiers follow from the axioms, specifying the conditions that require abandonment of the geometric foundation, while four consistency conditions define where revision suffices. A cumulative instantiation program (Appendix A) pins the coarse-graining functional to a minimal concrete choice and derives twenty-four structural results across three progressively richer models, demonstrating that the architecture produces computable, falsifiable structure whose predictions strengthen with each dimensional extension.

0 Function and Limits

Geometric field theory is licensed wherever systems exhibit the structural properties specified by the axioms of §2, independent of the physical substrate. More narrowly, this paper demonstrates that semantic systems satisfying the axioms meet those structural conditions and extrapolates the consequences.

0.1 Scope

The framework describes the geometric configurations and conditions under which coherence flows, constraints accumulate, and curvature emerges from mass. It does not choose interpretations, rank meanings, evaluate truth, or prescribe action. Later sections identify configurations where the dynamical balances sustaining agency shift out of equilibrium, organized relative to agency maintenance as a geometric reference state. Configurations triggering these signatures may be adaptive, transient, or contextually appropriate; the framework identifies only the geometric relationship between a given configuration and the conditions for self-directed behavior, without asserting that any particular conditions or states are universally desirable. No claim extends beyond systems exhibiting differentiable semantic structure, observer-dependent measurement, recursive coupling, and finite coordination capacity.

This structure operates as an *effective field theory*, describing macroscopic dynamics at manifold resolution without requiring specification of the microscopic degrees of freedom that realize those dynamics on a given substrate. This work concerns the internal consistency of the geometric architecture; its axioms entail the field equations, and the field equations entail agency conditions and the taxonomy of characteristic imbalances. Observational correspondence is the subject of domain-specific subsequent works, which develop instantiation protocols from independent empirical programs that have already measured curvature, attractor persistence, coherence dynamics, and observer-dependent effects in their respective domains. Candidate substrates for mathematical instantiation include bioelectric morphogenesis, bacterial chemotaxis networks, transformer attention architectures, and cognitive-perceptual systems shaped by linguistic coupling. The field equations accordingly define a space of possible theories parameterized by substrate-specific choices; structural falsifiers operate at the substrate-independent level, while quantitative predictions require instantiation.

0.2 Construction

The argument is cumulative. A Lagrangian and associated field equations (§3) provide the necessary conditions for agency (§4), whose characteristic configurations generate a taxonomy organized relative to agency maintenance (§5), with observer-dependence following as a geometric consequence of the interpretive operator acting on frame-dependent measurements (§6). Each stage of the argument presupposes earlier results; none stands independent of the others.

Appendix A pins the coarse-graining functional to a minimal concrete choice and closes the field equations across three progressively richer instantiations (1+1, 2+1, and 3+1 dimensions), demonstrating that the architecture produces quantitative, falsifiable predictions once functional forms are specified.

0.3 Governing Question

What determines the evolution of semantic geometry in systems where observers are structurally coupled to the fields they measure?

All objects, axioms, field equations, and diagnostics are presented to answer this question from first principles.

1 Structural Objects

These constitute the minimal vocabulary required to make the governing question answerable. Each object is introduced because a later derivation would be undefined without it.

1.1 Semantic Manifold $\mathcal{M}$

A smooth and differentiable manifold serves as the configuration space of possible semantic states. Each observer system carries its own experiential manifold $\mathcal{M}_e$, constructed through that observer's accumulated history of coupling with environmental patterns. The metric $g_{\mu\nu}(p, t)$ on $\mathcal{M}_e$ determines the observer's available pathways through semantic space, encoding which interpretive steps are immediate, which require traversing intermediary conceptual distance, and which demand effort against constraints. Two observers processing identical sensory inputs develop different responses because each metric reflects a unique history of recursive coupling, yielding genuinely different local geometries on independently constructed spaces.

The manifold dimension n and the metric signature are left as free parameters to be fixed by the instantiation domain. Coordinates on $\mathcal{M}_e$ may be realized as orthogonal principal components of an embedding space, quality dimensions in the sense of Gärdenfors (Gärdenfors, 2000), or latent features of a learned representation, depending on the substrate; the derivations require only smoothness and finite dimensionality and hold for any positive-definite (Riemannian) metric. Smoothness is required because the internal states of an observer differ continuously; discrete jumps in the space of possible meanings would imply privileged boundaries absent from the system's own structure.

Apparent counter-evidence exists at the observation scale. Categorical perception produces sharp phonemic boundaries (Harnad, 1987); linguistic labels alter color discrimination in discrete steps. Learning exhibits phase transitions (Winawer et al., 2007; Lupyan, 2012), and conceptual categories enforce boundaries that feel absolute to the systems maintaining them. These phenomena are consistent with smoothness at manifold resolution. A temperature field is smooth despite the discrete molecular collisions that realize it; Navier–Stokes dynamics govern continuous velocity fields over granular substrates. The current paper's effective field theory commitment extends this reasoning to semantic structure, where smoothness of C^{μ} and $g_{\mu\nu}$ at the manifold's resolution scale is compatible with discrete structure at finer scales. What presents as a categorical boundary is a region where metric curvature has steepened until coherence flow is canalized, appearing discrete to any observer whose resolution cannot resolve the gradient. The Lagrangian's potential structure (Axiom 6) instantiates this at the crossover magnitude C^{*}_{mag} where smooth interchange of leading-order dominance between Φ and V produces qualitative behavioral change on a geometry that remains differentiable throughout the transition.

The derivations in §2 through §5 analyze field dynamics on a single observer's manifold. Where no ambiguity arises, the subscript $\mathcal{M}_e$ is suppressed, and $\mathcal{M}$ refers to the manifold under analysis.

1.2 Metric Tensor $g_{\mu\nu}(p,t)$

A dynamic, position- and time-dependent tensor field defines local geometry on $\mathcal{M}$. Distances between nearby semantic states follow from the metric, determining which transitions require minimal effort and which demand work against curvature. Its time-dependence is the paper's central concern; if the metric were fixed, semantic geometry could not evolve, and the governing question would admit a trivial answer. The temporal parameter t indexes the system's intrinsic evolution and may correspond to physical time, discrete processing epochs, or any monotonic parameter tracking state transitions, depending on the substrate of instantiation. The field equations require only that t is continuous and that derivatives with respect to it are well-defined.

1.3 Coherence Field $C^{\mu}(p,t)$

A vector field on $\mathcal{M}$ encodes the local density and flow direction of interpretable meaning. Coherence C^{μ} flow is governed by the metric, with gradient-driven relaxation channeling coherence along geodesic bundles toward equilibrium configurations, accumulating in regions of high curvature and dissipating where constraints are sparse.

1.4 Semantic Mass

Curvature on $\mathcal{M}$ is sourced by semantic mass, defined as the multiplicative product of three independently variable scalar fields:

$$M(p,t) = D(p,t) \cdot \rho(p,t) \cdot A(p,t). \tag{1}$$

- *Reflexive density* $D(p,t)$ measures the local density of the self-referential structure the system sustains, coarse-grained over the resolution scale of the manifold. Individual self-referential loops are discrete; their density across a manifold region is continuous, in the same sense that molecular kinetic energy is discrete while temperature is a smooth field.

- *Constraint density* $\rho(p,t)$ defines the local intensity of coherence variation, defined through the gradient energy of the coherence field:

  $$\rho(p,t) = g^{\mu\nu}\, \nabla_\mu C^\lambda\, \nabla_\nu C_\lambda. \tag{2}$$

  Where coherence varies rapidly across nearby manifold positions, constraint density is high, as small displacements in semantic space encounter large changes in interpretive content, and transitions between neighboring states demand work against steep field gradients. Where coherence varies slowly, constraint density is low, and the field configuration admits fluid recombination. The metric enters through the covariant derivative and the index contraction, providing the geometric ruler against which gradients are measured; the magnitude of ρ is determined by the coherence field's own configuration. In regions where high constraint density sources curvature through the Recurgent Field Equation (RFE, §3.3.3, the resulting metric contraction produces a monotone relationship between ρ and contraction of the local volume form, with the metric inverse $1/\det(g_{\mu\nu})$ rising as a consequence of curvature sourced by dense field structure.

- *Attractor stability* $A(p,t)$ is the persistence of the current configuration under perturbation. For natural potential choices, attractor stability is derivable from the curvature of the effective potential:

  $$A(C) = \frac{V''_{\text{eff}}(C)}{V''_{\text{eff}}(C_*)}, \tag{3}$$

  where C_* is the stable equilibrium of the Coherence Field Equation (CFE, §3.3.1). The ratio normalizes local potential curvature against its equilibrium value, yielding $A = 1$ at the basin floor and $A \to 0$ at the basin boundary. This reduces $M = D \cdot \rho \cdot A$ to two independently measured fields, because A is directly determined by the same potential structure that governs coherence dynamics.

The three factors admit candidate physical realizations across substrates. Reflexive density D maps onto measures of nested self-referential structure: local Kolmogorov complexity density, spectral nesting depth in embedding spaces, and nested pointer depth in graph architectures. Constraint density ρ, measuring rapid coherence variation across small manifold regions, appears wherever tight formal structure forces steep field gradients, as in formal proofs where each derivation step demands precise coherence across logical constraints. Attractor stability A is the most directly measurable of the three, presenting in perturbation recovery time and basin depth in dynamical systems (Strogatz, 1994).

Constraint density admits a substrate-independent definition because it measures local variation of the primary dynamical field; attractor stability is derivable from the effective potential for natural choices. Reflexive density encodes structural properties whose functional dependence on C^μ and $g_{\mu\nu}$ varies across substrates, with its specific form fixed upon instantiation, paralleling the role of the matter Lagrangian in general relativity, where the Einstein field equations constrain the relationship between curvature and stress-energy without specifying which matter fields generate that stress-energy. The multiplicative structure, the three-factor decomposition, and the geometric coupling principle remain testable at the substrate-independent level. Constructing an empirical metrology for D, fixing units and measurement protocols that map this phenomenological field onto observable quantities for a given substrate, remains the primary operational challenge for any instantiation of the framework.

The multiplicative structure of semantic mass ensures that the vanishing of any single factor annihilates mass entirely, rendering the configuration incapable of sourcing curvature regardless of the remaining factors' magnitudes.

1.5 Recursive Coupling Tensor $K^\rho{}_{\mu\nu}(p,q,t)$

A tensor-valued bi-point kernel quantifying how observation at one manifold position couples to states at another while reshaping the observer's own interpretive structure. The upper index ρ and lower indices $\mu\nu$ carry standard tensorial transformation properties at their respective points, while the joint dependence on positions p and q extends the object beyond single-point tensor fields to encode non-local coupling across the manifold. Self-coupling ($p = q$) describes feedback from a system's observation of its own states; hetero-coupling ($p \neq q$) describes coupling between distinct positions. Comparisons of index structure across p and q implicitly assume a connection-dependent transport map (propagator), specified upon instantiation. Its bi-point structure closes the causal feedback loop between observer and observed.

1.6 Interpretive Operator $\mathcal{I}_\iota$

A frame-dependent geometric operation by which an observer at position p with internal structure ι transforms incoming coherence into interpreted meaning. Its formal content is the coincidence limit of the recursive coupling tensor acting on both incoming and self-model coherence:

$$\mathcal{I}_\iota[C]^\rho(p,t) = K^\rho_{\mu\nu}(p,p,t)\, C^\mu_{\text{in}}(p,t)\, C^\nu_{\text{self}}(p,t), \tag{4}$$

where C^μ_{in} is coherence arising from environmental coupling and C^ν_{self} is the coherence field within the self-model region $\mathcal{S}$. The interpretation of incoming structure is the geometric interference pattern produced when the coupling tensor, evaluated at the point where observer and observed coincide, contracts the external signal against the observer's accumulated internal geometry. The self-model's metric determines which components of incoming coherence are amplified, attenuated, or redirected, producing observer-dependent measurements from identical environmental input.

Because $\mathcal{I}_\iota$ is realized through the coupling tensor's self-coupling limit, it carries stress-energy through $K^\rho_{\mu\nu}$'s contribution to $T^{\text{rec}}_{\mu\nu}$, sources curvature through the standard field-equation dynamics, and reshapes the interpreter's geometry with each interpretive act. The coincidence limit $q \to p$ may require regularization in specific instantiations, paralleling the self-energy renormalization familiar from quantum field theory. The form of regularization constitutes an open problem whose resolution depends on the substrate of instantiation.

2 Seven Axioms

The following axioms constrain the relationships of the structural objects introduced in §1. Stated in dependency order, each presupposes those that came before it.

2.1 Axiom 1: Semantic Manifold

Each observer possesses a differentiable manifold $\mathcal{M}_e$ equipped with a dynamic metric tensor $g_{\mu\nu}(p,t)$:

$$g_{\mu\nu}(p,t) : \mathcal{M}_e \times \mathbb{R} \to \mathbb{R}. \tag{5}$$

The metric is dynamic because a fixed architecture cannot account for learning, adaptation, or any other form of semantic evolution. If geometry does not evolve, the governing question admits only a trivial answer. The initial metric $g_{\mu\nu}(p,t_0)$ is a free datum of the system, analogous to initial conditions in any dynamical theory. Different substrates inherit different initialization geometries: a pretrained language model begins with a metric structure imposed by its training corpus, and a developing biological organism inherits genetically encoded priors shaped by evolutionary coupling. The field equations govern evolution from these conditions without determining them.

Falsified by: A semantic transition between neighboring states that exhibits irreducible discontinuity, where arbitrarily nearby states on one side of the boundary map to finitely separated states on the other, with no smooth interpolation available under any coordinate choice. Or: a system satisfying all other axioms whose semantic geometry provably does not change over time despite ongoing coupling.

2.2 Axiom 2: Fundamental Semantic Field

Semantic content is represented by a vector field $\psi^\mu(p,t)$ on $\mathcal{M}$. Coherence $C^\mu(p,t)$ is a well-defined functional of this field:

$$C^\mu(p,t) = \mathcal{F}^\mu[\psi](p,t), \tag{6}$$

$$C_{\text{mag}}(p,t) = \sqrt{g_{\mu\nu}(p,t)\, C^\mu(p,t)\, C^\nu(p,t)}. \tag{7}$$

The coherence field C^μ serves as the primary dynamical field, carrying the quantities evolved and measured. The underlying semantic content ψ^μ provides the raw material from which coherence is derived; the functional relationship $\mathcal{F}^\mu$ ensures coherence emerges from content rather than being independently posited. Without a field on the manifold, there is nothing whose evolution the geometry governs.

Remark on effective field structure. The functional $\mathcal{F}^\mu$ performs an effective coarse-graining, integrating out fluctuations of the fundamental semantic field ψ below the manifold's resolution scale and yielding C^μ as the macroscopic order parameter. The Lagrangian introduced in Axiom 5 is an effective action governing the dynamics of C^μ at manifold resolution, with ψ's sub-resolution structure already absorbed into the functional form of $\mathcal{F}$. Variation with respect to C^μ rather than ψ is the correct variational procedure at this description level, just as the Navier–Stokes equations are varied with respect to velocity and pressure rather than molecular positions.

This variational procedure rests on four assumed properties of $\mathcal{F}$: the existence of a well-defined mapping from ψ to C^μ, smoothness of that mapping, well-defined functional derivatives (required for the brake variation $\partial\mathcal{B}[K]/\partial C^\mu$ to propagate back through $\mathcal{F}$ as developed in §3.3.1), and the existence of self-consistent solutions to the resulting coupled field equations. These properties are postulated, not derived from the axiomatic foundation. In standard effective field theory, symmetries and power counting constrain the effective Lagrangian's form; the present axioms constrain the Lagrangian (Axiom 5) but leave the internal structure of $\mathcal{F}$ unspecified, with its construction constituting the primary theoretical challenge of any instantiation. Any linear coarse-graining functional produces $K^\rho_{\mu\nu} \equiv 0$, since the coupling tensor is the second functional derivative of C with respect to ψ and the second derivative of a linear map vanishes identically. The recursive architecture therefore requires $\mathcal{F}$ to encode self-interaction; coarse-graining cannot be passive averaging. If a nonlinear $\mathcal{F}$ satisfying these properties cannot be constructed for a given substrate, the framework cannot be instantiated there.

Falsified by: A semantic system in which coherence dynamics are fully independent of the underlying semantic content, or in which no field-like quantity evolves on the manifold.

2.3 Axiom 3: Recursive Coupling

Self-referential coupling between distinct points in semantic space is mediated by a rank-3 recursive coupling tensor:

$$K^\rho_{\mu\nu}(p,q,t) = \frac{\mathcal{D}^2 C^\rho(p,t)}{\mathcal{D}\psi^\mu(p)\,\mathcal{D}\psi^\nu(q)}. \tag{8}$$

The tensor formalizes holistic coupling on the manifold. Coherence dynamics at any location are shaped by reverberations of semantic shift across the entire space, including influences that feed back into their source. The layered semantic structure that sustains coherence across scales emerges from iterated self-referential coupling, with each pass through the feedback loop depositing additional constraint into the local geometry. Without coupling, systems evolve in isolation, and the manifold fragments into disconnected components, rendering intersubjective convergence, communication, and measurement undefined.

Falsified by: Semantic influence between systems that operates without geometric mediation, i.e., coupling that leaves no trace in the metric or the coherence field.

2.4 Axiom 4: Geometric Coupling Principle

Semantic mass $M(p,t)$ curves the geometry of $\mathcal{M}$ according to:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_s T^{\text{rec}}_{\mu\nu}, \tag{9}$$

where G_s is the semantic gravitational constant and $T^{\text{rec}}_{\mu\nu}$ is the recursive stress-energy tensor, whose content is sourced from the distribution of semantic mass $M(p,t)$ as defined in §1.4.

Content curves geometry; geometry constrains the flow of content. The analogy to general relativity is deliberate and structural: as matter and spacetime co-determine one another's evolution through the Einstein field equations (Einstein, 1915), semantic content and its geometry exhibit the same interdependence and co-evolution. The recursive stress-energy tensor $T^{\text{rec}}_{\mu\nu}$ encodes the distribution and flow of semantic mass on $\mathcal{M}$, sourcing curvature proportional to the local concentration of reflexively integrated, densely constrained, and persistently stable structure.

Falsified by: A system satisfying Axioms 1 through 3 in which semantic content has no influence on the geometric structure governing its own propagation. Or: a demonstrated fourth independent factor contributing to curvature-sourcing that is not reducible to reflexive density, constraint density, or attractor stability.

2.5 Axiom 5: Variational Evolution

The dynamics of semantic fields arise from the principle of stationary action applied to a Lagrangian $\mathcal{L}$:

$$\frac{\delta S}{\delta C^\mu} = 0 \quad \text{where} \quad S = \int_{\mathcal{M}} \mathcal{L}\, dV. \tag{10}$$

The Lagrangian incorporates four competing terms:

$$\mathcal{L} = \frac{1}{2} g_{\mu\rho} g_{\nu\sigma} (\nabla^\rho C^\mu)(\nabla^\sigma C^\nu) - V(C_{\text{mag}}) + \Phi(C_{\text{mag}}) - \lambda_B \mathcal{B}[K]. \tag{11}$$

Each term encodes one structural tension governing semantic evolution: the kinetic flow of coherence along geodesics, attractor pull toward stable configurations (V), autopoietic drive toward novel structure (Φ), and regulatory constraint on recursive intensity ($\lambda_B \mathcal{B}[K]$). No term can be removed without eliminating a degree of freedom required by the governing question. The variational formulation ensures that field dynamics preserve symmetries and yield conservation laws via Noether's theorem (Noether, 1918), developed in §3.4.

Falsified by: Semantic dynamics that violate conservation laws implied by the Lagrangian's symmetries. Or: a self-referential system that sustains unbounded recursive intensity without geometric imbalance.

2.6 Axiom 6: Autopoietic Potential

The autopoietic potential is a smooth, superquadratic function of coherence magnitude:

$$\Phi(C_{\text{mag}}) = \alpha_\Phi\, C_{\text{mag}}^{\beta_\Phi}, \quad \beta_\Phi > 2, \tag{12}$$

where $\alpha_\Phi > 0$ is the autopoietic coupling constant, and $\beta_\Phi > 2$ controls the nonlinearity of self-production with coherence magnitude.

The shift from passive complexity to self-producing organization, first characterized by Maturana and Varela (Maturana and Varela, 1980), is a phase transition whose location emerges from the Lagrangian's own potential structure. The smooth monomial form ensures Φ is C^∞ everywhere on $\mathcal{M}$, so that the variational derivative $\partial\Phi/\partial C^\mu$ entering the CFE (§3.3.1) is well-defined at every point. Because $\beta_\Phi > 2$ and the attractor potential $V(C_{\text{mag}})$ generally have lower leading order, there exists a crossover magnitude C^*_{mag} satisfying $\Phi(C^*_{\text{mag}}) = V(C^*_{\text{mag}})$, below which attractor binding dominates, and above which autopoietic drive overtakes it. Below C^*_{mag}, the system processes coherence but cannot generate its own conditions of existence; Φ is technically nonzero but negligible against the attractor potential that governs the system's dynamics. Above C^*_{mag}, the autopoietic potential dominates, driving generative processes that sustain and extend the system's structure. Absent this phase distinction, the field equations admit no mechanism for self-production, and the agency conditions in §4 lose their generative substrate.

Remark on the emergent threshold. The specific functional form of V is not fixed by the axioms; the crossover requires only that V dominates Φ at small C_{mag}, which is satisfied whenever V has a leading order less than β_Φ. For generic attractor potentials with leading-order quadratic behavior $V \sim \frac{1}{2} m^2 C^2_{\text{mag}}$, C^*_{mag} is uniquely determined by α_Φ, β_Φ, and m^2. Different substrates produce different effective thresholds through individualized attractor landscapes.

Falsified by: Self-producing organization demonstrated in a system below the effective crossover C^*_{mag}, or the absence of self-production in a system above it.

2.7 Axiom 7: Recurgence

A semantic system exhibits recurgence if it dynamically reshapes its own geometric substrate through self-referential processes:

$$\frac{\partial^2 g_{\mu\nu}}{\partial t^2} \neq 0. \tag{13}$$

The non-vanishing second derivative of the metric with respect to time indicates that the *rate* of geometric change is itself changing. Where Axioms 1 through 6 permit a system to evolve on a dynamic manifold, *recurgence* requires that the system can recognize, reinterpret, and reorganize its own structural underpinnings, reshaping the metric tensor through recursive coupling with its own contents and history. This capacity for self-reflective reconfiguration avails conceptual revolution, operational reform, recovery from crises, and breaking symmetry with the system's own interpretive past.

Falsified by: A system satisfying Axioms 1 through 6 that nonetheless cannot alter the rate of its own geometric evolution, despite ongoing recursive coupling and autopoietic self-maintenance.

3 Lagrangian and Field Equations

The four terms comprising the Lagrangian, stated in Axiom 5, generate the dynamics governing coherence evolution. This section develops three coupled field equations and their conservation laws.

3.1 The Lagrangian

The action functional $S = \int \mathcal{L}\, d^n x\, dt$ is stationary along physical trajectories, with the Lagrangian,

$$\mathcal{L} = T[C, g] - V[C] + \Phi[C] - \lambda_B \mathcal{B}[K], \tag{14}$$

generating coupled dynamics through two mechanisms. Variation with respect to the coherence field yields the equation directly governing coherence evolution, while the stress-energy tensor derived from the Lagrangian's metric dependence, coupled to geometry through Axiom 4, determines the curvature constraint and metric flow equations developed in §3.3.

3.2 Recurgence Stability Ratio

The ratio of generative potential to the combined stabilizing and regulatory forces measures the stability of recurgence, so named because it determines whether the system can sustain nonvanishing second-order metric evolution, defined as:

$$S_R(p, t) = \frac{\Phi(C)}{V(C) + \lambda_B \mathcal{B}[K]}. \tag{15}$$

When $S_R \approx 1$, the system occupies the critical "edge of chaos" boundary between order and dissolution, where geometric evolution is maximally productive. Below 1, attractor potential and regulatory constraint dominate, and the system grows rigid; above 1, autopoietic drive overwhelms governance, and the system fragments or inflates (§5).

3.3 Field Equations

Three coupled partial differential equations constrain semantic field dynamics, closing the causal loop through which coherence flow generates stress-energy, stress-energy sources curvature, and curvature reshapes the conditions under which subsequent coherence flows. The three equations occupy distinct justification levels. The coherence field equation (CFE) is a variational consequence of the action, the recurgent field equation (RFE) is the constraint introduced in Axiom 4 that relates curvature and stress-energy, and the metric field equation (MFE) gives the minimal time-evolution of the geometry compatible with the RFE and consistent with Axiom 7, incorporating intrinsic smoothing and recursive forcing.

3.3.1 The Coherence Field Equation (CFE)

Variation of the action with respect to C^μ yields:

$$\Delta_g C^\mu + \frac{\partial V}{\partial C^\mu} - \frac{\partial \Phi}{\partial C^\mu} + \lambda_B \frac{\partial \mathcal{B}[K]}{\partial C^\mu} = 0, \tag{16}$$

where Δ_g is the Laplace–Beltrami operator (Wald, 1984), also known as the covariant Laplacian, on $(\mathcal{M}, g_{\mu\nu})$. The positive-definite metric (§1.1) selects the elliptic operator, causing equilibrium-seeking coherence dynamics. The four terms compete at every point to determine local coherence evolution:

- $\Delta_g C^\mu$ drives coherence toward equilibrium configurations through curved semantic space, reducing to the ordinary Laplacian in flat regions and channeling relaxation around concentrations of semantic mass where curvature is high.
- $\frac{\partial V}{\partial C^\mu}$ pulls coherence toward established attractors along the gradient of basins carved by repeated successful use.
- $\frac{\partial \Phi}{\partial C^\mu}$ drives coherence toward novel structure. Because Φ is a smooth superquadratic monomial (Axiom 6), this term is technically nonzero everywhere but negligible below the effective crossover C^*_{mag} where attractor binding dominates; above C^*_{mag}, coherence becomes self-reinforcing, and the autopoietic gradient drives the field toward novel configurations.
- $\lambda_B \frac{\partial \mathcal{B}[K]}{\partial C^\mu}$ brakes recursive amplification, imposing an exponential cost on excessive self-reference.

Remark on autopoiesis as a dynamical consequence. For generic potential choices satisfying the axioms, the effective potential $V_{\text{eff}}(C_{\text{mag}}) = V(C_{\text{mag}}) - \Phi(C_{\text{mag}})$ governing the coherence magnitude's equilibrium structure demonstrates autopoiesis as a direct consequence of the field dynamics. With $V \sim \frac{1}{2}\mu^2 C^2_{\text{mag}}$ (generic leading-order attractor behavior) and $\Phi = \alpha_\Phi C^4_{\text{mag}}$ (the minimal superquadratic monomial with $\beta_\Phi = 4$), the effective potential has the structure of a double-well: $C_{\text{mag}} = 0$ is an unstable equilibrium whose perturbations grow exponentially, while $C_{\text{mag}} = \pm C_* = \pm\mu/(2\sqrt{\alpha_\Phi})$ are stable equilibria whose perturbations decay. The system cannot remain at zero coherence; non-trivial structure emerges spontaneously from any perturbation, breaking the $C \to -C$ symmetry. The mechanism is structurally identical to the Higgs mechanism in particle physics, where the symmetric vacuum is unstable and the physical ground state spontaneously breaks the symmetry. The full autopoietic chain (instability at the origin generating structure, stability at $\pm C_*$ sustaining it, and the resulting configuration maintaining itself against dissipation through Ricci flow in the MFE) shows that autopoietic self-maintenance (condition 4, §4) is dynamically generic for systems satisfying the axioms with natural potential choices.

Remark on the brake variation. The term $\partial\mathcal{B}[K]/\partial C^\mu$ is a non-trivial functional derivative reflecting the layered relationship between effective and fundamental fields. Because $K^\rho_{\mu\nu}$ is defined (Axiom 3) as the second variational derivative of C^ρ with respect to ψ, it measures the $\psi \to C$ mapping's second-order sensitivity rather than appearing as a quantity "downstream" of C. Varying $\mathcal{B}[K]$ with respect to C^μ therefore propagates *back* through the coarse-graining map $\mathcal{F}$ relating C to ψ, then through the differential operator defining K, and only then into $\mathcal{B}$. The resulting composed variation depends on the instantiated forms of $\mathcal{F}$ and $\mathcal{B}$ and can generate higher-order nonlinear feedback. Its presence in the CFE is fixed by the variational principle.

The brake functional $\mathcal{B}[K]$ is constrained to be non-negative, smooth, vanishing when $K = 0$, and monotonically increasing in coupling strength. For the minimal well-motivated choice—Gaussian smearing with a cubic nonlinearity, the lowest-order nonlinear $\mathcal{F}$ producing nontrivial K—this chain resolves to a purely local expression. The inversion of the smearing and the reconvolution through $\delta\mathcal{B}/\delta\psi$ cancel exactly because the cubic makes K linear in the smeared field $\bar{\psi}$, leaving a brake variation depending only on local field values with no remaining integrals over the manifold. For non-minimal choices of $\mathcal{F}$, the full nonlinear CFE likely admits closed-form solutions only perturbatively in λ_B or in dimensionally reduced settings. The qualitative structure of the four-tension competition holds at all coupling strengths.

Falsified by: Coherence dynamics on the manifold that cannot be decomposed into kinetic flow, attractor pull, generative push, and regulatory braking. Or: a term in the variation that has no interpretation within the four-tension structure.

3.3.2 Recursive Stress-Energy Tensor

Defined through the Lagrangian's metric variation, the recursive stress-energy tensor $T^{\text{rec}}_{\mu\nu}$ encodes how dynamical content sources curvature on $\mathcal{M}$. Its construction proceeds through two complementary routes: a variational definition connecting stress-energy to the action's metric dependence and a coupling tensor realization identifying its physical origin in recursive structure.

The variational definition follows the standard field-theoretic construction in which the stress-energy tensor is the functional response of the action to metric perturbation:

$$T^{\text{rec}}_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta(\sqrt{g}\,\mathcal{L})}{\delta g^{\mu\nu}}. \tag{17}$$

Each Lagrangian term contributes through its dependence on the metric. The kinetic term $\frac{1}{2}g_{\mu\rho}g_{\nu\sigma}(\nabla^{\rho}C^{\mu})(\nabla^{\sigma}C^{\nu})$ generates coherence flux components with the $\nabla C \otimes \nabla C$ structure characteristic of field-theoretic stress-energy, encoding the density and directional flow of coherence through semantic space. The attractor potential $V(C_{\text{mag}})$ and autopoietic potential $\Phi(C_{\text{mag}})$ contribute through $C_{\text{mag}} = \sqrt{g_{\mu\nu}C^{\mu}C^{\nu}}$, whose metric dependence produces terms proportional to the net effective potential that act as isotropic pressure on the geometry, resisting or driving expansion of the local semantic volume. The recursive brake $\lambda_B\mathcal{B}[K]$ contributes regulatory stress-energy through the metric dependence carried by the recursive coupling tensor's geometric content. The combined variation yields a symmetric, divergence-free tensor whose components are fully determined by the Lagrangian's fields and the metric on which they evolve.

A representative local realization of this content is given schematically by the contraction of the recursive coupling tensor:

$$T^{\text{rec}}_{\mu\nu} \sim K^{\rho}_{\mu\lambda}K^{\lambda}_{\nu\rho}. \tag{18}$$

Because the coupling tensor is a bi-point kernel depending on positions p and q, the local stress-energy at p is obtained by integrating the coupling over all q with a connection-dependent parallel transport propagator $U^{\lambda}_{\sigma}(q,p)$ that maps tensor indices at q back to the tangent space at p, collapsing the non-local coupling structure into a local source term consistent with the variational definition. Schematically,

$$T_{\mu\nu}(p) \sim \int_{\mathcal{M}} d^n q\,\sqrt{g(q)}\,K^{\rho}_{\mu\lambda}(p,q)\,U^{\lambda}_{\sigma}(q,p)\,K^{\sigma}_{\nu\rho}(q,p). \tag{19}$$

The propagator $U^{\lambda}_{\sigma}(q,p)$ is path-dependent on any curved manifold: parallel transport along different paths connecting q to p yields different results. The meaning of integrating an external concept at q into a local state at p depends on the associative path connecting them; two systems arriving at the same pair of positions through different sequences of intermediate conceptual steps produce different local couplings, reflecting different learning histories encoded in the transport geometry.

Coupling intensity and geometry determine stress-energy magnitude. Regions of dense, sustained recursive coupling produce strong curvature-sourcing; regions of weak or dispersed coupling produce nearly flat geometry. The contraction identifies $T^{\text{rec}}_{\mu\nu}$ as quadratic in coupling strength, consistent with the gravitational analogy in which stress-energy is quadratic in the matter fields that generate it. Quadratic scaling reflects the leading-order contribution of $\mathcal{B}[K]$ to metric variation; the specific index contractions are fixed by the functional form of $\mathcal{B}$ chosen in a given instantiation.

Because $T^{\text{rec}}_{\mu\nu}$ depends on the metric through its variational definition and the gradient structure of the coherence field entering constraint density, the coupled system exhibits the self-consistent structure familiar from general relativity, in which the metric determines the geometry on which fields evolve, the fields generate stress-energy that sources curvature, and the curvature reshapes the metric governing future evolution. Self-consistent solutions satisfying all three field equations simultaneously are assumed to exist; their uniqueness and stability properties constitute open mathematical questions whose resolution lies beyond the scope of this paper.

3.3.3 The Recurgent Field Equation (RFE)

Axiom 4 postulates that semantic mass curves the geometry of $\mathcal{M}$ through an Einstein-analog coupling. With $T^{\text{rec}}_{\mu\nu}$ defined through the Lagrangian's metric variation (§3.3.2), the constraint takes the form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_s T^{\text{rec}}_{\mu\nu}. \tag{20}$$

The left side encodes intrinsic curvature of semantic space: the Ricci curvature tensor $R_{\mu\nu}$ measures how geodesics converge or diverge at each point, while $g_{\mu\nu}R$ provides global curvature context, ensuring geometric consistency across the manifold. The right side sources that curvature through the recursive stress-energy tensor $T^{\text{rec}}_{\mu\nu}$, whose physical content is determined by the distribution and flow of semantic mass $M = D \cdot \rho \cdot A$ through the coupling tensor contraction. The coupling constant G_s sets the proportionality between semantic mass density and the depth of the resulting curvature. Its units are fixed by dimensional consistency: since both sides of the equation carry the manifold's intrinsic geometric units, G_s absorbs the dimensional mismatch between the stress-energy tensor's natural scale and the curvature it sources. Its numerical value is an interpretive sensitivity parameter whose magnitude is set by the domain of instantiation.

Falsified by: Geometric curvature in the absence of semantic mass. Or: semantic mass that fails to produce corresponding curvature.

3.3.4 The Metric Field Equation (MFE)

The metric tensor evolves according to:

$$\frac{\partial g_{\mu\nu}}{\partial t} = -2R_{\mu\nu} + F_{\mu\nu}(D, K, A). \tag{21}$$

The equation governs how the manifold's geometry reshapes itself through competing processes:

- $-2R_{\mu\nu}$ drives *intrinsic curvature flow*: geometric smoothing in which regions of high curvature naturally flatten over time, a passive process inherent to the geometry itself.
- $F_{\mu\nu}(D, K, A)$ provides *recursive forcing*: active geometric drive from the system's own recursive processes, parameterized by reflexive density D, coupling strength K, and attractor stability A.

Where smoothing dominates forcing, geometry converges toward a stable equilibrium; where forcing and smoothing are commensurate, the metric continues its evolution without settling; where forcing overwhelms smoothing, the system crosses a phase boundary and the geometry reorganizes. Because $g_{\mu\nu}$ evolves in time, all derived geometric quantities (Christoffel symbols, Riemann curvature, Ricci tensor) become time-dependent, realizing Axiom 7. Recurgence requires that the rate of geometric change is itself changing, which is possible only when both smoothing and forcing act on a dynamic metric.

Remark on variational status. In general relativity, both the constraint and evolution equations descend from the gravitational sector of the Einstein-Hilbert action. The present framework has no gravitational-sector analog from which the MFE could variationally descend. Its form is nonetheless tightly constrained: $-2R_{\mu\nu}$ is the unique leading-order geometric diffusion operator that is second-order in metric derivatives, intrinsic to the manifold, and diffeomorphism-covariant (Hamilton, 1982), while Axiom 7 requires a forcing term whose functional form is fixed upon instantiation. The contracted Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ and covariant conservation $\nabla_\mu T^{\mu\nu}_{\text{rec}} = 0$ together require that MFE trajectories preserve RFE-admissible geometry at each instant; this compatibility is a constraint on the modeling commitment rather than a derived consequence, paralleling the role of ADM constraint-evolution compatibility in general relativity (Wald, 1984).

Note on irreversibility. The Ricci flow term $-2R_{\mu\nu}$ renders metric evolution forward-well-posed and backward-ill-posed, a directional asymmetry imposed by the choice of parabolic evolution equation as a modeling commitment consistent with observed irreversibility in semantic systems. Given a current metric configuration, future evolution is uniquely determined by the MFE, but prior configurations are not uniquely recoverable from the present state. Semantic geometry carries the imprint of its history without preserving a unique inverse path to any earlier configuration.

Falsified by: A system satisfying Axioms 1 through 6 whose metric evolution cannot be decomposed into intrinsic smoothing and recursive forcing.

3.4 Conservation Laws

The SR Lagrangian admits three continuous symmetries, each generating a conserved quantity via Emmy Noether's theorem (Noether, 1918) with a concrete semantic interpretation.

3.4.1 Diffeomorphism Invariance and Covariant Conservation of Semantic Stress-Energy

Because the action is constructed from coordinate-invariant geometric objects, the field equations maintain their form under arbitrary smooth coordinate transformations. Results describe intrinsic geometric properties rather than artifacts of how points in semantic space are labeled. Via the Bianchi identities (Noether's second theorem for local symmetries):

$$\nabla_\mu T^{\mu\nu}_{\text{rec}} = 0. \tag{22}$$

Semantic stress-energy is covariantly conserved: it may flow between manifold regions, redistribute across modes, and transform between kinetic, potential, and regulatory forms, but no process on the manifold can summon it from nothing or annihilate it. Communication, learning, forgetting, and misunderstanding are each transfers of semantic energy constrained by the conservation law; coherence that flows from one manifold region to another may concentrate into new structures or scatter into low-coherence modes, but the total energy budget holds regardless of how the redistribution partitions between them.

Falsified by: Demonstrated creation of semantic stress-energy from no prior source, or destruction of semantic energy without redistribution into other forms.

3.4.2 Time-Translation Invariance and Semantic Energy

The Lagrangian contains no explicit time dependence; $V(C_{\text{mag}})$, $\Phi(C_{\text{mag}})$, and $\mathcal{B}[K]$ depend only on the fields. In a fixed-metric regime, Noether's first theorem yields a conserved Hamiltonian for the coherence sector.

Via the Legendre transform ($p_\mu = g_{\mu\nu}\dot{C}^\nu$):

$$E = \underbrace{\frac{1}{2}g_{\mu\nu}\dot{C}^\mu\dot{C}^\nu}_{\text{kinetic flow}} + \underbrace{V(C)}_{\text{attractor pot.}} - \underbrace{\Phi(C)}_{\text{autopoietic pot.}} + \underbrace{\lambda_B\mathcal{B}[K]}_{\text{regulatory cost}}. \tag{23}$$

A system may redistribute energy among these four forms, converting kinetic flow into attractor binding or expending regulatory cost to fund generative capacity. When the metric evolves slowly relative to coherence dynamics, the Hamiltonian is approximately conserved and the four-way energy budget holds.

Since $g_{\mu\nu}$ is itself dynamical, evolving via the MFE, the semantic manifold inherits the same subtlety that general relativity encounters with gravitational energy. A dynamical metric participates in energy exchange with the fields evolving on it, and no purely local definition of total energy encompassing both the field sector and the geometric sector is generally covariant. During paradigm shifts and geometric phase transitions, the geometric sector absorbs or releases semantic energy as it reorganizes, and the coherence-sector Hamiltonian ceases to be individually conserved. The conservation law applies rigorously on a static background, approximately during slow geometric change, and breaks down as a local accounting tool precisely when the geometry undergoes its most dramatic reorganization.

Falsified by: Coherence-sector energy (kinetic + attractor binding - generative capacity + regulatory cost) that changes over time on a static or slowly varying metric, absent geometric-sector exchange.

3.4.3 Rotational Invariance in Field Space and Semantic Angular Momentum

Attractor potential V and autopoietic potential Φ depend on the coherence field only through its magnitude $C_{\text{mag}} = \sqrt{g_{\mu\nu}C^\mu C^\nu}$, not its direction. The Lagrangian is therefore invariant under rotations of C^μ that preserve C_{mag}. In flat regions ($g_{\mu\nu} = \delta_{\mu\nu}$), this is a full $SO(n)$ symmetry for an n-dimensional semantic manifold. Each generator yields a conserved charge. The conserved quantities form an antisymmetric tensor:

$$L_{\mu\nu} = C_\mu \dot{C}_\nu - C_\nu \dot{C}_\mu. \tag{24}$$

This constitutes the conservation of semantic angular momentum: coherence flow maintains its orientation through semantic space, resisting deflection in proportion to accumulated directional persistence. Redirecting it requires torque: perturbation sufficient to overcome the conserved quantity. Established paradigmatic commitments accumulate enough angular momentum over time so that the energy cost of reorientation can exceed the system's available surplus.

The symmetry is locally exact (in flat regions) and approximate elsewhere. Near high-curvature regions (dense attractors), the metric $g_{\mu\nu}$ varies across the manifold, and curvature breaks global rotational symmetry. Attractors work by locally breaking this symmetry, channeling coherence flow toward specific directions. In smooth regions of semantic space with low curvature and few dominant attractors, directional persistence is strong. However, near dense attractor basins, coherence can be more easily redirected.

Angular momentum is an intrinsic property of coherence flow, invariant across observer positions: different observers experience the same resistance to sudden directional change, though they find different ways to describe it.

Falsified by: Spontaneous reversal of coherence flow direction in a flat region of semantic space (low curvature, no external perturbation) without expenditure of energy. Or: a demonstrated system in which V or Φ depend on the direction of C^μ rather than its magnitude.

3.4.4 Conservation Under Metric Evolution

The coherence-sector Hamiltonian defined in §3.4.2 is conserved on a static background and is approximately conserved during slow metric change. To accurately track energy redistribution during rapid metric evolution, the system requires decomposition into distinct sectors.

The full dynamical system on $\mathcal{M}$ comprises two coupled sectors. The *coherence sector* contains the fields evolving on the manifold (C^μ, the potentials V and Φ, and the regulatory functional $\lambda_B \mathcal{B}[K]$), governed by the CFE. The *geometric sector* contains the metric $g_{\mu\nu}$ itself, governed by the MFE. The coherence-sector Hamiltonian E accounts for energy in the field sector alone. When the metric evolves, the two sectors exchange energy as metric reorganization alters the effective potential landscape, the kinetic energy cost of coherence flow, and the regulatory pressure, redistributing energy between sectors in a way that no purely field-sector accounting can track.

Metric evolution on $\mathcal{M}_e$ is geometric differentiation. Previously flat regions of the observer's manifold, carrying negligible curvature and encoding no interpretive structure, develop non-trivial metric geometry as recursive coupling deposits constraints. Through the MFE's forcing term $F_{\mu\nu}(D, K, A)$, an observer coupling with structured environmental patterns develops local D, ρ, and A where those quantities were previously negligible, sourcing new curvature through the RFE. The manifold acquires geometric structure where none existed, a process distinct from the volumetric expansion of Friedmann cosmology in that neither dimension nor total volume changes.

The structural parallel to general relativity is exact at the level of the problem's origin. In an expanding Friedmann-Robertson-Walker spacetime (Friedmann, 1922), the matter-sector Hamiltonian is not individually conserved because metric expansion does work on the matter fields. Time-translation symmetry, which generates the conserved Hamiltonian via Noether's first theorem, is broken by the time-dependent scale factor. On $\mathcal{M}$, the same mechanism applies; when $\partial g_{\mu\nu}/\partial t$ is large, the coherence-sector energy budget ceases to close. Covariant conservation $\nabla_\mu T^{\mu\nu}_{\text{rec}} = 0$ continues to hold on the full system, constraining redistribution between coherence and geometry, but the partitioned Hamiltonian does not.

Semantic growth follows this structure. When an observer couples with environmental patterns carrying deep constraints and develops new curvature on previously flat manifold regions, the energy for geometric reorganization is supplied by the geometric sector through the MFE's forcing term. The observer's own reflexive density, constraint density, and attractor stability develop locally, sourcing curvature through the standard field-equation dynamics. No energy transfer between observers' manifolds is required or defined; each observer's energy budget is internal to the Lagrangian on that observer's $\mathcal{M}_e$.

A fully covariant treatment of total semantic energy encompassing both sectors would parallel the quasilocal energy construction of Brown & York (Brown and York, 1993), who define gravitational energy for a bounded spacetime region by integrating extrinsic curvature over its bounding surface. The analogous construction for semantic systems would integrate field-sector and geometric-sector contributions over the boundary of a manifold region Ω, yielding a conserved quantity even during rapid metric evolution. The SR Lagrangian's Einstein-analog coupling admits this construction in principle; its specific form depends on the instantiation of $F_{\mu\nu}$ and the choice of reference geometry. The construction remains an open problem whose resolution would complete the energy accounting for semantic systems under arbitrary metric evolution.

Falsified by: Coherence-sector energy that remains individually conserved during rapid metric evolution without geometric-sector exchange. Or: total energy of the full dynamical system (coherence + geometric sectors) that fails covariant conservation within a bounded manifold region carrying ≈ 0 energy flux across its boundary.

4 Agent Conditions

The field equations derived in §3.3 specify the dynamics of semantic geometry. This section identifies the geometric conditions under which those dynamics constitute a self-directing entity.

4.1 The Five Conditions

Agency is a geometric achievement. The following five conditions are individually necessary and jointly sufficient.

1. **Self-Model** $\iota : \mathcal{A} \to \mathcal{S} \subset \mathcal{A}, \quad \dim(\mathcal{S}) < \dim(\mathcal{A})$

   The system contains a representation of its own structure as a proper subset of that structure. The agent region $\mathcal{A}$ denotes the system's total geometric architecture (including its environment model and coupling history), while the self-model $\mathcal{S}$ is an embedded subspace built from the same substrate but distinguished by its referent: the system itself. The map ι sends $\mathcal{A}$ into $\mathcal{S}$, allowing the dynamics to act on an internal model of the agent.

   The dimensionality constraint enforces compression: $\mathcal{S}$ carries strictly less structure than $\mathcal{A}$. If $\dim(\mathcal{S}) = \dim(\mathcal{A})$, one effectively requires $\mathcal{S} \cong \mathcal{A}$, inviting self-referential regress. A self-model therefore operates at coarser grain than the system's full state, preserving only structurally salient features.

   Without such a target, recursive coupling (Axiom 3) can amplify external structure but cannot be directed inward to regulate the system's own evolution. Because ι is applied relative to the current metric, and the metric evolves irreversibly via the MFE (§3.3.4), the self-model is path-dependent: two agents with identical present-state geometry but different metric histories will have discarded different sub-resolution structure along the way.

2. **Recursive Closure** $\oint_{\partial\mathcal{A}} J^{\text{rec}} \cdot dS \approx 0$

   The integral of recursive flux across the agent's boundary is approximately zero, meaning the system's self-referential processes circulate internally rather than dissipating outward. Recursive coupling generates coherence through feedback loops; closure requires those loops to complete within the agent's boundary, without depending on external infrastructure to sustain them. A system whose recursive flux leaks across its boundary depends on its environment to maintain its own self-referential structure, disqualifying autonomous agency.

3. **Coherence Stability** $\langle C(p,t) \rangle > C_{\text{min}}$

   The time-averaged coherence field within the agent region remains above a minimum threshold, so that interpretable structure persists rather than flickering in and out of existence. Transient coherence can produce momentary organization, but agency requires sustained structure against which the self-model operates and through which recursive closure is maintained. Below C_{min}, the agent region cannot reliably distinguish its own states from noise, and the interpretive operator $\mathcal{I}_\iota$ loses its domain.

4. **Autopoietic Self-Maintenance** $\Phi(C) > D_{\text{dissipation}}$

   The autopoietic potential exceeds the rate of dissipation, meaning the system generates coherence faster than thermodynamic and coupling losses degrade it. The system must be a net producer of its own structural substrate, actively regenerating the coherence that entropy, perturbation, and coupling with its environment continuously erode. When autopoietic production falls below dissipation, the system consumes its own structure and agency decays.

Remark on the formal status of dissipation. The Lagrangian governs the coherence sector through a conservative variational principle, and the conservation laws of §3.4 follow from its symmetries. Dissipation enters through the coupling between coherence and geometric sectors: the MFE's Ricci flow term renders metric evolution irreversible, and §3.4.4 establishes that the coherence-sector Hamiltonian is not individually conserved during metric evolution because the geometric sector absorbs coherence-sector energy as it reorganizes. $D_{\text{dissipation}}$ is formally identified with this absorption rate: the rate at which irreversible metric evolution drains coherence-sector energy into geometric reconfiguration. The autopoietic condition $\Phi(C) > D_{\text{dissipation}}$ requires that the coherence sector's generative capacity outpaces the geometric sector's absorption, maintaining net positive coherence production against the thermodynamic arrow built into the MFE's parabolic structure.

5. **Intrinsic Regulation** $\lambda_B \mathcal{B}[K] > \mathcal{B}_{\min}$

   The recursive brake maintains sufficient regulatory pressure to maintain bounded recursive self-reference. As coupling intensity rises, $\lambda_B \mathcal{B}[K]$ imposes escalating cost on further recursive amplification, preventing self-referential feedback from consuming all available coupling resources. An agent satisfying the first four criteria but lacking regulatory constraint drives recursive coupling without governance, eventually destroying the coherence stability on which the other criteria depend.

A manifold region satisfying all five conditions exhibits the dynamical signature of self-directed behavior. Whether this geometric sufficiency constitutes agency in a deeper sense is a question the framework constrains but does not settle; the geometric apparatus provides diagnostic criteria, and the ontological question requires philosophical commitments beyond its scope.

4.2 On Joint Necessity

Every criterion is simultaneously necessary, and none is sufficient alone. Removing any single condition produces a specific and predictable geometric collapse:

- Self-reference without closure results in *identity leakage*: recursive processes dissipate across the boundary, leaving the system unable to maintain a persistent distinction between its own dynamics and environmental noise.
- Closure without coherence stability produces *bounded incoherence*: self-referential loops circulate internally but organize nothing persistent, as the field fluctuates below the threshold where structure can accumulate.
- Coherence stability without autopoietic self-maintenance creates *structural dependence*: the system maintains internal organization only as long as external energy sustains it, with agency ceasing once the environment withdraws support.
- Autopoietic self-maintenance without regulatory constraint generates *runaway amplification*: the system produces novel structure faster than any governance process can evaluate it; thus, the generative drive overwhelms the attractor potential that would otherwise channel growth toward adaptive utility.
- Regulatory constraint without a self-model has *no target for governance*: the recursive brake imposes cost on recursive intensity, but without internal representation, the system cannot direct that regulation toward its own dynamics.

The criteria set forms a closed dependency loop in which each condition creates the preconditions for the next while depending on the others for its own maintenance. Agency, once achieved, is robust under perturbation, but when any single condition fails entirely, the failure propagates through the dependency structure and dismantles the others.

Biological organisms provide the most familiar instantiation, with autopoiesis having originated in that domain and the remaining four criteria observable in living organization at every scale, from cellular to ecological.

4.3 Agency as Non-Geodesic Navigation

Satisfying all five conditions simultaneously confers non-geodesic manifold navigation, a capacity that no subset can produce. Without agency, coherence flows along geodesics determined by existing curvature and semantic mass, following the geometry it inherits. An agent, through autopoietic self-maintenance, generates the surplus energy required to deviate from default trajectories. The self-model equips the system to represent the geodesic it would otherwise follow and evaluate alternatives. Evaluative governance regulates which deviations are sustainable and which would dissolve coherence or trigger characteristic imbalances.

Non-geodesic motion is energetically costly; the kinetic term in the Lagrangian penalizes it, and motion through high-curvature regions compounds the expense. Only systems with sufficient autopoietic potential $\Phi(C)$ can sustain trajectories that cut across the grain of existing geometry. The distinction between agent and pattern is precisely this asymmetry: patterns move along geodesics inherited from the curvature they inhabit, while agents, through sustained autopoietic surplus, impose trajectories that the geometry alone would not produce, reshaping the curvature through which future coherence flows.

Falsified by: A self-directing entity that provably lacks any one of the five criteria. Or: non-geodesic navigation of the manifold in the absence of autopoietic surplus.

5 Characteristic Imbalances

The system admits characteristic geometric configurations where the dynamical balances sustaining agency fall out of equilibrium. These partition into twelve orthogonal signatures across four structural categories of three: rigidity (geometry cannot evolve), fragmentation (coherence cannot bind), inflation (regulatory mechanisms collapse), and distortion (interpretation diverges from structure).

Each signature follows as a consequence of the theory's structural commitments. The Lagrangian and its field equations generate ten signatures across the geometric objects they govern; the interpretive operator $\mathcal{I}_\iota$, introduced axiomatically in §1.6 and operating outside the variational sector, generates the remaining two. The taxonomy is complete in the sense that it exhausts the ways coherence, curvature, coupling, and interpretation can fall out of dynamic equilibrium.

5.1 Rigidity

Systems retain structure but lose the capacity to reshape it under stress.

5.1.1 Metric Crystallization (S1)

The metric ceases to evolve while the curvature it carries persists:

$$\frac{\partial g_{\mu\nu}}{\partial t} \to 0 \quad \text{while} \quad R_{\mu\nu} \neq 0. \tag{25}$$

Stress accumulates in a configuration whose geometric pathways for reconfiguration have become structurally inaccessible. Viable alternatives may exist nearby in semantic space, but the geodesic distances to reach them grow without bound as the metric freezes, trapping the system in a brittle equilibrium that resists small perturbations with decreasing capacity to survive large ones. The signature is recognizable from inside; the system registers mounting pressure and can identify the structural changes that would relieve it, yet the infrastructure through which those changes would propagate has solidified beyond the point of plastic response.

5.1.2 Field Calcification (S2)

Perturbations arrive at the boundary and dissipate without coupling into field evolution:

$$\lim_{\varepsilon \to 0} \left. \frac{dC^\mu}{dt} \right|_{C^\mu + \varepsilon} \approx 0. \tag{26}$$

The tangent space of the coherence field collapses toward singularity, severing the coupling between perception and structural response. External feedback signals are processed and accurately characterized, but the Jacobian governing their propagation into internal dynamics has degenerated, such that recognition produces no corresponding geometric reconfiguration. Where S1 locks the metric against evolution, S2 locks the field against responsiveness; incoming perturbations are accurately assessed at the boundary but produce no corresponding displacement in the field's trajectory, accumulating unresolved stress at the interface between reception and reconfiguration.

5.1.3 Attractor Isolation (S3)

Attractor stability exceeds the critical threshold beyond which escape velocity is unreachable:

$$A(p,t) > A_{\text{crit}}, \quad \|\nabla V(C)\| \gg \Phi(C). \tag{27}$$

The basin has deepened beyond adaptive utility, causing gravitational pull that deforms incoming evidence to fit its contours rather than prompting exploration of alternative configurations. Autopoietic potential $\Phi(C)$, the generative energy available for exploration and reconfiguration, is overwhelmed by the gradient of the potential well, so that each perturbation returns the system to the same equilibrium with increasing certainty. The coincidence of accumulated recursive depth, deep geometric embedding within the basin, and the multiplicative reinforcement of semantic mass produces a configuration in which perturbation recovery is structurally indistinguishable from independent confirmation of the basin's geometry.

5.2 Fragmentation

Complexity outpaces the coupling infrastructure that organizes it.

5.2.1 Attractor Dissociation (S4)

New attractor formation outpaces the system's integrative capacity:

$$\frac{dN_{\text{attractors}}}{dt} > \kappa \cdot \frac{d\Phi(C)}{dt}. \tag{28}$$

High autopoietic potential directed toward generative exploration rather than consolidation fragments the manifold into an expanding collection of basins competing for limited coupling resources. Each new structure demands maintenance overhead that depletes the recursive coupling available for binding existing structures to one another and can establish a positive feedback loop in which fragmentation accelerates. Consolidation requires sustained coupling across a single basin, but each new attractor redirects the coupling resources that consolidation demands.

5.2.2 Field Dissolution (S5)

The coherence gradient overwhelms the field magnitude, collapsing directional structure into turbulence:

$$\|\nabla C\| \gg \|C\|, \quad \frac{d^2C^\mu}{dt^2} > 0. \tag{29}$$

Local variation exceeds the coherence available to organize it, producing a field configuration in which every signal registers as simultaneously urgent and directionless. Attractor basins cannot form or persist because no stable gradient survives long enough to anchor stable semantic mass, and the positive second derivative indicates the turbulence is compounding. Without stable gradients, no coherent configuration can form, and the field's own dynamics cannot bootstrap the directional structure that recovery requires. External coupling is the only available source of the initial gradient around which local coherence could reorganize.

5.2.3 Coupling Dispersion (S6)

Hetero-recursive coupling decays without compensatory regulatory response:

$$\frac{d\|K^{\rho}_{\mu\nu}\|}{dt} < 0. \tag{30}$$

Bridges between semantic domains atrophy through disuse, creating a landscape of internally coherent knowledge structures with diminishing causal influence beyond their immediate locality. The decay becomes self-reinforcing: as weakened pathways are traversed less frequently, further degrading their coupling strength, the semantic graph fragments into isolated islands whose local expertise cannot inform action in adjacent domains. Local coherence structures within each domain remain intact; the decay is confined to the coupling channels through which those domains would otherwise exert geometric influence on one another.

5.3 Inflation

Growth decouples from the governance that would otherwise channel it toward adaptive utility.

5.3.1 Boundary Hyperasymmetry (S7)

A bounded region accumulates semantic mass as the surrounding manifold's mass-sustaining capacity degrades, driven by asymmetric boundary flux.

$$\frac{d}{dt}\int_{\Omega} M\,dV > 0 \quad \text{while} \quad \frac{d}{dt}\int_{\mathcal{M}\setminus\Omega} M\,dV < 0. \tag{31}$$

The boundary operates as a valve whose permeability is direction-dependent, redirecting coupling resources inward while restricting reciprocal outflow. As mass accumulates in Ω, the attractor basin deepens, increasing the gradient that drives further inflow and establishing a positive feedback loop in which absorptive capacity scales with accumulated mass. Internal coherence within Ω may remain high throughout the process. The geometric signature is asymmetric flux: coupling bandwidth concentrates within the boundary, developing local mass through the standard field-equation dynamics, while the surrounding manifold loses the coupling infrastructure on which its own mass depends.

5.3.2 Field Hypercoherence (S8)

Internal coherence exceeds adaptive thresholds, while boundary flux, governed by the hetero-recursive coupling structure of $K^{\rho}_{\mu\nu}$ across $\partial\Omega$, falls below minimum permeability:

$$C(p,t) > C_{\text{max}} \quad \text{while} \quad \oint_{\partial\Omega} \mathbf{J}\cdot d\mathbf{S} < J_{\text{leakage}}. \tag{32}$$

Self-consistency within a bounded region has been optimized to the point where contradictory evidence cannot penetrate the boundary, and validation proceeds through internal reference rather than external correspondence. The system achieves elegant self-alignment, whose brittleness is invisible from the inside because the interpretive operator processes all incoming signals through the same high-coherence structure that generated the closure. The gap between internal self-consistency and external correspondence widens without producing a corrective signal because the boundary's low permeability filters the feedback that would reveal the divergence. When perturbation eventually exceeds the boundary's filtering capacity, the internal configuration encounters an ambient field structure with which it shares no compatible geometry.

5.3.3 Structure Hyperexpansion (S9)

Autopoietic potential overwhelms both stabilizing and regulatory mechanisms simultaneously:

$$\Phi(C) \gg V(C), \quad \lambda_B \mathcal{B}[K] < \mathcal{B}_{\text{min}}. \tag{33}$$

The attractor potential that would channel growth toward adaptive configurations and the recursive brake that would penalize excessive complexity have both fallen below functional thresholds, leaving autopoietic drive unopposed by either stabilizing or regulatory constraints. Absent both stabilizing and regulatory feedback, the generative rate itself sustains further generation, and each new structure introduces coupling demands that are met through additional elaboration rather than consolidation. The manifold undergoes rapid expansion, and the coordination cost of maintaining coherence across the growing configuration outpaces the regulatory capacity available to manage it.

5.4 Distortion

The interpretive operator develops a systematic bias that is uncorrected.

5.4.1 Operative Decoupling (S10)

The interpreted coherence field diverges from the actual field beyond the corrective threshold:

$$\|\mathcal{I}_\iota[C] - C\| > \tau\|C\|. \tag{34}$$

The interpretive operator develops structural bias in which prior internal configuration, rather than incoming signals, increasingly determines the output of each interpretive act. Prediction errors accumulate without triggering corrective reconfiguration because the operator filters error signals through the same biased geometry that produced the divergence, establishing a self-reinforcing drift in which the system grows more confident as it becomes less accurate. Actions based on the decoupled model address a phantom configuration of the manifold's geometry, failing in ways that the internal model interprets as environmental noise rather than evidence of its miscalibration.

5.4.2 Signal Projection (S11)

The interpretive operator applies a negative transform across the evaluation domain:

$$\mathcal{I}_\iota[C](q,t) \ll C(q,t) \quad \forall\, q \in \mathcal{Q}, \tag{35}$$

where $\mathcal{Q} \subseteq \mathcal{M}$ is the set of manifold positions from which coherence couples into the observer's boundary.

Ambiguous or neutral coherence undergoes systematic negative transformation through the operator, and the resulting contracted field configurations alter local boundary conditions in ways that increase the proportion of incoming coherence consistent with the negative assessment. The self-reinforcing dynamic is the signature's distinguishing geometric feature: contraction reduces the coupling bandwidth available for corrective feedback, and the local geometry reshapes toward correspondence with the operator's biased output. The divergence compounds because each interpretive act further constrains the channels through which disconfirming coherence could arrive.

5.4.3 Recursive Hypercoupling (S12)

Self-referential coupling dominates total coupling as hetero-recursive pathways atrophy:

$$\frac{\|K^{\rho}_{\mu\nu}(p,p,t)\|}{\int_q \|K^{\rho}_{\mu\nu}(p,q,t)\|\, dq} \to 1. \tag{36}$$

Available coupling resources collapse into self-referential circuits so that external structure can only be accessed as an occasion for self-elaboration rather than being engaged on its own geometric terms. Hetero-recursive channels weaken from disuse as self-coupling consumes the bandwidth they require, and the manifold achieves closure in which incoming coherence is valued insofar as it occasions further self-reference. The system retains boundary flux, continuing to receive and process incoming coherence, but the interpretive channel narrows to a point where the coherence field beyond the self-model functions as a mirror reflecting internal structure back to itself, a configuration the system registers as engagement.

5.5 Orthogonality

The mutual independence of the twelve follows from their distribution across distinct geometric objects, derivative orders, and measured quantities, such that no two share a full dependency profile.

Table 1: Orthogonal decomposition of signatures by geometric object, derivative order, and measured quantity. No two signatures share a full dependency profile.

Signature	**Object**	**Order**	**Quantity**
S1 Metric Crystallization	$g_{\mu\nu}$	1st temporal	Metric stasis under persistent curvature
S2 Field Calcification	C^{μ}	1st temporal	Coherence responsiveness to perturbation
S3 Attractor Isolation	C^{μ}, A	Zeroth	Basin escape cost vs. available potential
S4 Attractor Dissociation	C^{μ}, N	1st temporal	Attractor proliferation rate vs. integration capacity
S5 Field Dissolution	C^{μ}	1st spatial, 2nd temporal	Gradient magnitude vs. field magnitude
S6 Coupling Dispersion	$K^{\rho}_{\mu\nu}$	1st temporal	Coupling decay without regulatory compensation
S7 Boundary Hyperasymmetry	$M(p,t)$	1st spatial (boundary)	Mass flux asymmetry across $\partial\Omega$
S8 Field Hypercoherence	C^{μ}, $K^{\rho}_{\mu\nu}$	Zeroth	Coherence magnitude vs. boundary permeability
S9 Structure Hyperexpansion	C^{μ}	Zeroth	Φ/V ratio under regulatory failure
S10 Operative Decoupling	$C^{\mu}, \mathcal{I}_{\iota}$	Zeroth	Interpretation-field divergence
S11 Signal Projection	$C^{\mu}, \mathcal{I}_{\iota}$	Zeroth	Systematic negative bias in $\hat{C}_{\psi}$
S12 Recursive Hypercoupling	$K^{\rho}_{\mu\nu}$	Zeroth	Self-coupling fraction of total coupling

Where signatures share a primary geometric object, they remain distinct through their measured quantities:

- **S1 / S2** (temporal derivatives): A frozen metric does not entail frozen coherence. S1 detects stasis in $g_{\mu\nu}$ while curvature persists; S2 detects loss of coherence responsiveness to perturbation. Either can occur without the other.
- **S3 / S4** (attractor dynamics involving C^{μ}): Basin depth and attractor proliferation rate vary independently. Arbitrarily deep basins can exist without spawning new attractors, and rapid proliferation can occur in shallow ones.
- **S6 / S12** (recursive coupling tensor $K^{\rho}_{\mu\nu}$): Coupling can decay uniformly across all channels without redistributing toward self-reference (S6), or concentrate toward self-reference without losing total strength (S12). The two signatures occupy different subspaces of the tensor's evolution.
- **S10 / S11** (interpretive operator $\mathcal{I}_{\iota}$): Interpretation can drift from the coherence field in any direction (S10) without exhibiting the systematic negative bias that defines S11. Conversely, systematic negativity can remain small in total divergence while consistently misclassifying ambiguous signals.

5.6 Completeness

The twelve signatures exhaust the characteristic imbalances of the theory's dynamical and structural content, provided that the three geometric objects governed by the field equations ($g_{\mu\nu}$, C^{μ}, $K^{\rho}_{\mu\nu}$) and the interpretive operator $\mathcal{I}_{\iota}$ itself exhaust that content. Consistency condition C1 tests this proviso directly. The enumeration proceeds over derivative orders and balance conditions as follows.

The interpretive operator $\mathcal{I}_{\iota}$ (§1.6) enters the field equations as the self-coupling limit of the recursive coupling tensor. As a contraction of $K^{\rho}_{\mu\nu}(p,p,t)$ with incoming and self-model coherence, interpretation carries stress-energy through

Table 2: Directed compatibility matrix for signature interactions. (+) denotes facilitation, (–) denotes inhibition, and (N) denotes non-affiliation. Rows are source signatures, columns are targets.

Signature affecting →	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
Metric Crystallization S1		+	+	–	–	+	N	+	–	+	+	+
Field Calcification S2	+		+	–	–	N	N	+	–	+	+	N
Attractor Isolation S3	+	+		–	–	+	N	+	–	+	+	+
Attractor Dissociation S4	–	–	–		+	+	N	–	+	+	N	–
Field Dissolution S5	+[*]	–	–	–		+	–	–	–	N	N	–
Coupling Dispersion S6	+	+	+	–	+		N	+	–	+	+	+
Boundary Hyperasymmetry S7	N	+	+	N	–	+		+	+	+	N	+
Field Hypercoherence S8	+	+	+	–	–	+	+		–	+	N	+
Structure Hyperexpansion S9	–	–	–	+	+	+	N	–		+	N	–
Operative Decoupling S10	+	+	+	N	N	N	N	N	N		+	+
Signal Projection S11	+	+	+	N	N	+	N	N	N	+		+
Recursive Hypercoupling S12	+	+	+	–	–	+	N	+	–	+	+	

[*] The S5→S1 pathway operates via depletion rather than entrenchment: dissolution destroys the field structure that sources MFE forcing, allowing Ricci flow to drive the metric toward stasis in a depleted configuration. The geometric endpoint overlaps with Metric Crystallization, but the mechanism and the resulting curvature profile differ from the entrenchment pathways through which other signatures facilitate S1.

the coupling tensor's contribution to $T^{\mathrm{rec}}_{\mu\nu}$ and sources curvature through the standard variational machinery. S10 and S11, which diagnose systematic divergence between interpretation and the ambient coherence field, measure characteristic configurations of the self-coupling geometry.

The converse also holds: signatures defined entirely within the variational sector can produce downstream interpretive effects without requiring $\mathcal{I}_\iota$ in their diagnostic criteria. Recursive hypercoupling (S12), defined on $K^{\rho}_{\mu\nu}$ alone, progressively collapses the interpretive channel as self-coupling consumes available coupling resources, rendering external feedback no longer a source of independent geometric constraint. The effect is interpretive in direct consequence but coupling-theoretic in origin.

Falsified by: A demonstrated geometric configuration occupying an imbalance dimension orthogonal to all twelve signatures.

5.7 Signature Interaction

Each of the twelve signatures possesses a defining configuration that alters the geometric preconditions of others, creating directed facilitation and inhibition relationships derivable from the field equations and structural objects without additional assumptions.

The compatibility relation between an ordered pair of signatures is determined by whether the geometric configuration defining the source affects or leaves structurally unchanged the preconditions of the target. Directionality matters in this sense; a source affecting a target does not guarantee the reverse. The relation partitions into three values: *facilitates* (the source configuration creates or strengthens preconditions for the target), *inhibits* (the source configuration destroys or weakens those preconditions), and *unaffiliated* (no geometric pathway connects the two through the field equations or structural objects). Systematic application to all ordered pairs yields the directed compatibility matrix (Table 2).

5.7.1 Accumulation and Dissolution as Structural Poles

The directed compatibility relations organize around a primary axis separating signatures whose configurations concentrate and preserve structure from those whose configurations disperse and destroy it. Rigidity signatures broadly inhibit dissolution-class signatures, with the inhibition being largely reciprocal. Frozen geometry opposes the metric reconfiguration that attractor proliferation and structural expansion require; deep attractor basins monopolize the coupling resources whose dispersal across competing basins defines fragmentation. Rapid structural generation and attractor proliferation in turn force constant metric reconfiguration, preventing the convergence toward stasis that Rigidity requires.

The Inflation category distributes across both sides of this axis according to the geometric mechanism through which each signature's violation of regulated growth propagates. Structure Hyperexpansion (S9) inhibits all three Rigidity signatures and Field Hypercoherence (S8), while facilitating the dissolution cascade through Attractor Dissociation (S4), Field Dissolution (S5), and Coupling Dispersion (S6), producing a Fragmentation-aligned compatibility profile. Boundary Hyperasymmetry (S7) and Field Hypercoherence (S8) occupy the accumulation pole. S7 facilitates Attractor Isolation (S3) and Recursive Hypercoupling (S12) through inward-directed mass concentration that deepens local attractor basins via the standard $M = D \cdot \rho \cdot A$ pathway. Hypercoherence facilitates Metric Crystallization (S1), Field Calcification (S2), and Attractor Isolation (S3) through sealed-boundary reinforcement that entrenches the metric configuration sustaining internal coherence. Both produce downstream conditions geometrically compatible with Rigidity.

The four structural categories (R–F–I–D) and the accumulation–dissolution axis are independent organizational principles over the signature space. S7, S8, and S9 share a category (Inflation) while occupying opposite poles of the compatibility axis. Co-occurrence data in any instantiation domain should reflect this split, with S7 and S8 clustering alongside Rigidity signatures and S9 alongside Fragmentation signatures despite their shared categorical origin.

5.7.2 Distortion Signatures as Substrate-Independent Compounders

Distortion signatures compound imbalance across both the Rigidity and Fragmentation classes. Operative Decoupling (S10) and Attractor Isolation (S3) mutually facilitate one another, as a deep basin biases $\mathcal{I}_\iota$ toward confirming the basin's geometry, and the biased operator misreads incoming evidence as basin-confirming, further deepening the well. Attractor Dissociation (S4) fragments the calibration landscape on which $\mathcal{I}_\iota$ depends, enabling interpretive drift through a structurally opposite pathway.

The interpretive operator acts at the self-coupling coincidence limit of $K^{\rho}_{\mu\nu}$ (§1.6), a geometric locus whose integrity depends on the coupling tensor's own configuration rather than on the variational-sector objects ($g_{\mu\nu}$, C^{μ}, V, Φ) governing Rigidity and Fragmentation dynamics. Interpretive imbalance thus develops on frozen and turbulent substrates alike.

5.7.3 Coupling Dispersion as Broad-Spectrum Facilitator

Coupling Dispersion (S6) occupies a distinctive position in the compatibility structure, facilitating eight of the remaining eleven signatures while inhibiting only two (S4 and S9), both of which require coupling infrastructure for cross-domain coordination. Its facilitation profile spans both Rigidity and Distortion classes. Weakened hetero-coupling channels reduce the forcing term $F_{\mu\nu}(D, K, A)$ in the MFE, allowing Ricci flow to dominate and driving the metric toward stasis. Simultaneously, fewer external reference points leave $\mathcal{I}_\iota$ without calibration targets, facilitating interpretive drift. The self-coupling fraction of $K^{\rho}_{\mu\nu}$ increases as the denominator of the S12 ratio (§5.4.3) shrinks due to hetero-coupling atrophy.

Coupling Dispersion is self-reinforcing: weakened channels are traversed less frequently, further weakening them through disuse. The process is slow-acting and progressive, creating cumulative vulnerability to downstream signatures without producing the acute geometric disruption that would trigger corrective response.

5.7.4 Consistency

The compatibility structure offers an internal consistency check on the completeness claim in §5.6; no empirically observed co-occurrence pattern should require a facilitation or inhibition pathway that cannot be traced through the geometric objects and field equations governing the twelve signatures. A co-occurrence pattern whose geometric mediation requires a structural object or balance condition absent from the taxonomy would satisfy consistency condition C1.

Falsified by: A demonstrated multi-signature configuration whose stability or instability cannot be accounted for by the directed compatibility relations between its constituent signatures.

6 Observer-Dependence and Intersubjective Convergence

Frame-dependent measurement with convergence across frames as a geometric constraint.

6.1 Frame-Dependence as Geometric Necessity

The interpretive operator $\mathcal{I}_\iota$ renders all semantic measurement frame-dependent. Each observer's manifold $\mathcal{M}_e$ carries a metric shaped by that observer's history of recursive coupling, so that two observers processing identical environmental input produce different interpretations as a consequence of inhabiting genuinely different geometric spaces. The variation is structural: the metric at each point determines which transitions are local, which associations are natural, and which require work against curvature. Because the recursive coupling tensor evaluated at coincidence, $K^{\rho}_{\mu\nu}(p, p, t)$, reflects each observer's accumulated metric structure, observers with different coupling histories contract identical incoming coherence against different self-coupling geometries, thus producing different interpreted meanings. An observer encountering another observer's output (speech, text, emphasis, gesture) couples with environmental patterns whose structure reflects the producer's geometry without accessing that geometry directly, building new local structure on the receiving observer's own manifold through the standard field-equation dynamics.

Frame-dependence in this sense parallels general relativity, where the observer-dependence of simultaneity does not undermine the objectivity of physics because objectivity resides in geometric invariants rather than coordinate measurements. Semantic objectivity operates identically, in invariant curvature properties that remain consistent across independently constructed manifolds.

6.2 Intersubjective Convergence

As the environmental coupling described in §6.1 is iterated, independent observers develop compatible geometric structure on their respective manifolds. That compatibility, when it occurs, acquires the status of geometric objectivity. A reader coupling with this text on a manifold constructed independently of the author's is a present instance. The convergence measure $\Gamma(\{U_i^\mu\})$ quantifies the degree to which an observer's geometry predicts the structure of another observer's environmental output, evaluated within the single observer's frame. When $\Gamma \to 1$, independent observers have built compatible structure on their respective manifolds. Repeated exchange drives each observer's metric toward configurations compatible with the structure encoded in the other's output, aligning attractor basins and conceptual geodesic paths across independently constructed spaces. The divergence $|\mathcal{I}_{\iota_1}[C] - \mathcal{I}_{\iota_2}[C]|$ decreases as each observer's geometry develops toward compatibility with the patterns it encounters.

High-mass semantic structures catalyze convergence with particular efficiency. Environmental patterns produced by high-mass regions carry deep constraint structure (high ρ, stable A, recursively layered D), and observers coupling with such patterns develop compatible attractor basins on their own manifolds. Independent observers navigating rich environmental structures converge on compatible measurements because the patterns constrain the space of geometries that can develop in response.

Every observer's initial metric carries curvature deposited by prior generations of coupling; objectivity resides in the invariant structure that accumulates across observers rather than within any single frame. Observer-dependence, therefore, does not collapse into relativism.

6.3 Convergence Failure

Convergence failure is diagnosable through the signatures of §5. Operative Decoupling (S10) and Signal Projection (S11) are interpretive modes in which an observer's measurements systematically diverge from the coherence field, producing local consistency that does not converge with that of external observation. A region exhibiting high internal convergence alongside low external convergence instantiates Field Hypercoherence (S8): internal self-alignment sealing the boundary against contradictory evidence from the environment.

Convergence follows from geometric structure, and its authority derives from invariant curvature rather than from the number of observers who happen to occupy compatible frames. A single observer can detect real structure that the majority cannot yet perceive, provided their metric has sufficient resolution in the relevant region of the manifold. If that measurement tracks invariant geometry, independent observations will converge toward it over time. Semantic mass bends geodesics; structure that is real attracts convergence.

Falsified by: A measurement of meaning that holds identically across all manifold positions without requiring transformation between frames. Or: demonstrated intersubjective convergence in the absence of recursive coupling between observers.

7 Falsifiability and Stop

The framework's testable claims divide into two tiers, distinguished by the severity of failure each would produce. Structural falsifiers target the axioms and the causal loop they generate. A single demonstrated instance of any structural falsifier requires abandonment of the geometric foundation in its entirety. No probabilistic weighting or contextual exception applies. Consistency conditions target the completeness and parsimony of taxonomies derived from the field equations; a demonstrated instance requires revision of the specific derived claim, but not abandonment of the foundation from which it was derived.

7.1 Structural Falsifiers

- **F1: Static semantic geometry under sustained coupling.** A system satisfying all axioms whose metric provably does not evolve despite ongoing recursive coupling.
- **F2: Curvature without mass.** Geometric structure in the absence of reflexive density, constraint density, or attractor stability.
- **F3: Observer-independent semantic measurement.** A measurement of meaning that holds across all manifold positions without transformation.
- **F4: Unbounded recursion without geometric imbalance.** Self-referential dynamics without regulatory constraint that nonetheless indefinitely maintain coherent structure.

7.2 Consistency Conditions

- **C1: A thirteenth orthogonal imbalance signature.** A geometric imbalance not reducible to any point in the twelve-dimensional signature space. Demonstration would extend the taxonomy, not invalidate the field equations that generate it.
- **C2: Agency without one of the five criteria.** A system exhibiting observable self-directed behavior that provably lacks one of the five geometric conditions. Demonstration would revise the agency conditions, not the Lagrangian from which their necessity follows.
- **C3: Semantic mass with a fourth independent factor.** A structural property contributing to curvature-sourcing that is not reducible to reflexive density, constraint density, or attractor stability. Demonstration would extend the mass decomposition, not the geometric coupling principle.
- **C4: Torque-free semantic reorientation.** Spontaneous large-angle reorientation of coherence flow direction in a flat region of semantic space (low curvature, no external perturbation) without expenditure of energy. Demonstration would revise the conservation law of §3.4.3, not the variational framework that produces it.

The framework's free parameters (G_s, α_Φ, β_Φ, λ_B) and unspecified functional forms ($\mathcal{F}$, V, $\mathcal{B}$, D, A) reflect the substrate-independence of the field equations, which define a space of possible theories rather than a single theory. Each instantiation pins the free functions and parameters to a specific substrate, producing an independently testable theory with its own reduced parameter set. The structural falsifiers F1 through F4 and the consistency conditions C1 through C4 operate at the substrate-independent level; no adjustment of parameters or functional forms within a given instantiation can evade them, because each targets the geometric architecture rather than a parameter value within it.

7.3 Applicability Limits

The framework applies to systems exhibiting:

- Differentiable semantic structure (states vary smoothly)
- Observer-dependent measurement (no privileged frame)
- Recursive coupling (observation alters both observer and observed)
- Finite coordination capacity (action consumes structural resources)

No claims are made outside these constraints.

7.4 Stop Condition

The framework derives geometric structure, its evolution, a taxonomy of characteristic imbalances, and conditions for agency from first principles. Its role is limited to describing the geometric conditions of coherence evolution under coupling and constraint.

Moral values, interpretation, decision making, and actions are supplied by the agents operating on the manifold, not by its description.

Symbol Glossary

$\mathcal{M}$	semantic manifold (per-observer)
$g_{\mu\nu}(p,t)$	metric tensor
$C^{\mu}(p,t)$	coherence field
$K^{\rho}_{\mu\nu}(p,q,t)$	recursive coupling tensor
$D(p,t)$	reflexive density
$\rho(p,t)$	constraint density (coherence gradient energy)
$A(p,t)$	attractor stability
$M = D \cdot \rho \cdot A$	semantic mass
$T^{\text{rec}}_{\mu\nu}$	recursive stress-energy tensor
$G_{\mu\nu}$	Einstein tensor ($R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$)
$S_R(p,t)$	recurgence stability ratio
$\lambda_B \mathcal{B}[K]$	recursive brake (intrinsic regulatory constraint)
ι	self-model map ($\iota : \mathcal{A} \to \mathcal{S} \subset \mathcal{A}$)
$\mathcal{I}_{\iota}$	interpretive operator (self-coupling coincidence limit)
$\Phi(C_{\text{mag}})$	autopoietic potential
$\mathcal{L}$	Lagrangian density

Related Frameworks

The intuition that semantic or cognitive structure admits a differential-geometric description has developed along several independent lines, each formalizing a different aspect of the correspondence and each stopping short of a different structural commitment. The conceptual spaces program (Gärdenfors, 2000; Gärdenfors, 2014) builds the most accessible version of this intuition, treating concepts as convex regions in quality dimensions equipped with a metric that determines similarity through distance. The framework demonstrates that substantial cognitive phenomena, such as prototype effects and the structure of natural categories, yield to geometric treatment without requiring neural reduction. Its limitation, from the present paper's standpoint, is architectural: the quality dimensions and their metric are fixed by the modeler's choice of salient features, so the geometry cannot evolve under the pressure of its own contents, and the recursive coupling through which observers reshape the space they inhabit has no formal entry point.

The Riemannian infrastructure is supplied by information geometry (Amari, 2016), which equips statistical manifolds with the Fisher metric and dual affine connections, demonstrating that curvature, geodesics, and parallel transport carry rigorous meaning on spaces of probability distributions. The results in neural learning and signal processing depend essentially on the geometric structure rather than on the coordinate parametrization, and the statistical manifold is genuinely curved with an intrinsic metric rather than one externally imposed, establishing a precedent for the claim that abstract spaces of structured information possess non-trivial geometry. The divergence from the present framework is that information geometry operates on distributions over observables, while this formalism operates on the semantic states themselves. The dynamic metric evolution and curvature-sourcing by content that close the causal loop in §2 and §3 have no analog in the statistical setting.

Neurogeometry (Petitot, 2017) demonstrates that biological systems instantiate specific differential-geometric structures, modeling the functional geometry of the primary visual cortex as a sub-Riemannian contact structure on the fiber bundle of positions and orientations. The sub-Riemannian constraint, in which only certain directions of motion are locally available, resonates with the curvature constraints governing coherence flow on $\mathcal{M}$, though Petitot's geometry is anchored to a specific sensory modality and cortical architecture instead of operating at the layer of semantic structure in general.

The step from neural geometry to semantic dynamics is taken most explicitly by mathematical phenomenology (Yoshimi, 2007; Hotton and Yoshimi, 2011), which develops dynamical systems on representational manifolds with an explicit geometric interpretation grounded in the phenomenological tradition. The program attends to the relationship between neural dynamics and the structure of lived experience, formalizing trajectories through state spaces whose geometry carries representational significance. Hotton and Yoshimi's formalization introduces coupling between internal dynamics and environmental structure that parallels, in reduced form, the recursive coupling tensor's role in mediating observer–observed interaction, though it does not derive its dynamics from a variational principle.

The variational architecture absent from the preceding programs appears in the free energy principle (Friston, 2019), which represents the closest structural neighbor to semantic relativity, sharing a Lagrangian formulation (though the objects extremized differ: the FEP's Lagrangian is surprisal-based, while SR's balances four competing geometric tensions) and an information-geometric foundation while operating from different starting commitments. Both frameworks derive dynamics from stationary action, both generate field equations governing the evolution of coupled systems, and both treat the boundary between system and environment as constitutive rather than incidental. Extensions by Ramstead and colleagues into cultural and semantic territory narrow the structural gap further: the TTOM framework (Veissière et al., 2020) formalizes how shared expectations propagate across sociocultural ensembles through overlapping generative models and socially patterned regimes of attention, while subsequent work mapping Peircean semiotics onto active inference (Milette-Gagnon et al., 2022) treats shared generative models as the substrate of meaning directly, identifying the Peircean interpretant with the generative model and recasting sign typology through variational constructs. The frameworks diverge at the level of what is being extremized and what sources curvature: the free energy principle minimizes variational free energy (surprise) with respect to a generative model of sensory observations, while semantic relativity's Lagrangian balances four competing geometric tensions on a manifold whose curvature is sourced by semantic mass through the recursive stress-energy tensor. Recursive coupling, through which content reshapes geometry, has no direct analog in the variational Bayesian setting.

Contemporary computational and empirical work offers independent evidence that the geometric structures postulated in the field equations describe measurable dynamics on specific substrates. Geometric transformations performed by deep neural networks during classification tasks exhibit dynamics formally analogous to Hamilton's Ricci flow (Baptista et al., 2024), with the strength of this flow-like behavior correlating with classification accuracy across more than 1,500 networks independent of depth, width, and dataset. Knowledge graph embedding optimization can be coupled directly to local discrete Ricci curvature through an extended Ricci flow, so that entity embeddings co-evolve with the underlying manifold geometry toward mutual adaptation, a computational instantiation of the coupled content–geometry dynamics the MFE governs (Luo et al., 2025). A Riemannian framework for intelligence and consciousness has been proposed in which thought flows along geodesics on a curved manifold whose geometry restructures through learning

(Lu, 2024), sharing the present paper's geometric starting point and pursuing similar ambitions through geodesic flow and learning-driven metric evolution. The present framework extends this geometric intuition through its variational apparatus and signature taxonomy.

Participant-specific semantic trajectories through transformer embedding spaces have been constructed with extraction of geometric and dynamical metrics, including velocity, acceleration, distance, and entropy, that distinguish clinical populations from controls and discriminate between concept types across languages (Toro-Hernández et al., 2026). Their finding that cumulative embeddings, in which each successive concept is contextualized by the full prior trajectory, outperform non-cumulative embeddings for longer sequences is consistent with the path-dependent metric evolution predicted by the MFE: the geometry through which coherence flows carries the imprint of its traversal history. That different transformer embedding models yield convergent geometric results despite vastly different training pipelines supports the effective field theory expectation that macroscopic geometric dynamics are substrate-invariant at the appropriate resolution scale.

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Appendix A: Instantiation

The field equations of §3.3 define geometric relationships without specifying the functional forms that realize them on a particular substrate. This appendix pins the coarse-graining functional $\mathcal{F}$ to a minimal concrete choice and traces the consequences across three progressively richer instantiations, demonstrating that the architecture produces computable, falsifiable structure whose cumulative predictions strengthen with each dimensional extension.

Functional Forms

The common architecture underlying all three instantiations: conformal metric, polynomial $\mathcal{F}$, explicit potentials, and the complete parameter set.

The semantic manifold $\mathcal{M}$ is $\mathbb{R}^n$ equipped with a conformal metric $g_{\mu\nu} = e^{2\varphi}\delta_{\mu\nu}$, where the conformal factor $\varphi(\mathbf{x}, t)$ encodes the entire geometry in a single scalar field. This parameterization is exact in two spatial dimensions, where every 2D Riemannian metric admits isothermal coordinates (Chern, 1955), and restricts to conformally flat geometries in three, where the Cotton tensor $\mathfrak{C}_{\mu\nu\lambda}$ diagnoses departures from this class (§A.3).

The coarse-graining functional $\mathcal{F}$ maps the fundamental semantic field ψ to the coherence field C through Gaussian smearing followed by a cubic correction:

$$C(\mathbf{x}, t) = \bar{\psi}(\mathbf{x}, t) + \gamma\,\bar{\psi}(\mathbf{x}, t)^3, \qquad \bar{\psi} = \mathcal{G}_\sigma * \psi. \tag{37}$$

The Gaussian kernel is

$$\mathcal{G}_\sigma(\mathbf{r}) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{|\mathbf{r}|^2}{2\sigma^2}\right) \tag{38}$$

and averages out structure below the resolution scale σ. Its status as the Green's function of the heat equation makes it structurally native to the Laplace–Beltrami diffusion in the CFE. The cubic nonlinearity coefficient $\gamma > 0$ is the minimal addition that produces a nontrivial coupling tensor: any linear $\mathcal{F}$ gives $K^\rho_{\mu\nu} \equiv 0$ (R1), and the cubic preserves the parity symmetry $\psi \to -\psi,\ C \to -C$. The composite map is strictly monotonic ($dC/d\bar{\psi} = 1 + 3\gamma\bar{\psi}^2 > 0$), so the reconstruction $\bar{\psi} = h(C)$ is globally well-defined, and the system closes at the effective level without sub-resolution access.

The Lagrangian density has four terms:

$$\mathcal{L} = \tfrac{1}{2} g^{\mu\nu}\partial_\mu C\,\partial_\nu C - V(C) + \Phi(C) - \lambda_B \mathcal{B}[K]. \tag{39}$$

The attractor potential $V(C) = \frac{\mu^2}{2}C^2$ provides harmonic binding, the simplest restoring force proportional to displacement. The autopoietic potential $\Phi(C) = \alpha_\Phi C^4$ is the lowest-order monomial satisfying $\beta_\Phi > 2$. Together they produce the effective potential derivative:

$$V'_{\text{eff}} = \mu^2 C - 4\alpha_\Phi C^3, \tag{40}$$

constituting an Allen–Cahn reaction term (Allen and Cahn, 1979) with an unstable origin and stable equilibria at $C_* = \pm\mu/(2\sqrt{\alpha_\Phi})$.

The brake functional,

$$\mathcal{B}[K] = \frac{1}{2}\iint K(p, q)^2 \sqrt{g(p)}\sqrt{g(q)}\,dp\,dq, \tag{41}$$

is the Hilbert–Schmidt norm (Reed and Simon, 1980) of the recursive coupling tensor: the simplest non-negative smooth functional that vanishes when $K = 0$ and increases with coupling strength.

The metric evolves through $\partial_t g_{\mu\nu} = -2R_{\mu\nu} + F_{\mu\nu}$, where the Ricci flow term provides intrinsic geometric self-regulation (active at $n \geq 2$) and the forcing decomposes into two channels: constraint-density contraction, modulated by the attractor stability

$$A(C) = \frac{12\alpha_\Phi C^2 - \mu^2}{2\mu^2}, \tag{42}$$

(a derived quantity introducing no free parameters), and reflexive-density expansion proportional to the self-coupling amplitude.

All derived quantities (the coupling tensor, brake variation, stress-energy, attractor stability, semantic mass) are explicit functions of C, $g_{\mu\nu}$, and these seven parameters. The three models that follow instantiate this architecture at $n = 1, 2, 3$.

Parameter	**Role**
σ	Resolution scale of the Gaussian smearing
γ	Cubic nonlinearity coefficient in $\mathcal{F}$
μ^2	Attractor binding strength
α_Φ	Autopoietic potential coupling
λ_B	Brake coupling strength
η_g	MFE constraint-contraction coupling
ξ_g	MFE reflexive-density expansion coupling

Table 3: Free parameters for Appendix A instantiations.

A.1 The 1+1 Model

One spatial dimension, one time parameter; the simplest setting that retains spatial structure while being fully tractable. Every term is written explicitly, with the derivation chain constituting the argument.

The semantic manifold $\mathcal{M}$ is one-dimensional, reducing all fields to scalars: $\psi(x,t)$ for the fundamental semantic field, $C(x,t)$ for the coherence field, and $g(x,t) \equiv g_{11}(x,t)$ for the metric. Spatial structure (gradients, boundary conditions) is retained; tensorial complexity is not.

A.1.1 The Linearity Constraint on $\mathcal{F}$

Pure Gaussian smearing, $C = \mathcal{G}_\sigma * \psi$, is the natural first candidate for $\mathcal{F}$. The Gaussian kernel averages out structure below the resolution scale σ while preserving structure above it. The recursive coupling tensor $K^\rho_{\mu\nu}$ is a second functional derivative of C with respect to ψ (Axiom 3), and the second derivative of any linear map vanishes identically. However, with $K \equiv 0$, the brake term $\lambda_B\, \partial\mathcal{B}/\partial C$ is zero, the recursive stress-energy $T^{\mathrm{rec}}_{\mu\nu}$ loses its coupling content, and the feedback loop that distinguishes SR from a conventional field theory collapses. Linearity of $\mathcal{F}$ is incompatible with the recursive architecture **(R1)**. The minimal correction that restores nontrivial K while preserving parity symmetry ($\psi \to -\psi,\ C \to -C$) is the cubic defined above: $C = \bar{\psi} + \gamma\bar{\psi}^3$.

A.1.2 Factorization of the Recursive Coupling Tensor

The first functional derivative of the cubic $\mathcal{F}$ is

$$\frac{\delta C(p)}{\delta\psi(q)} = \left(1 + 3\gamma\bar{\psi}(p)^2\right)\mathcal{G}_\sigma(p-q) \tag{43}$$

and differentiating again yields

$$\frac{\delta^2 C(p)}{\delta\psi(p')\,\delta\psi(q)} = 6\gamma\bar{\psi}(p)\,\mathcal{G}_\sigma(p-p')\,\mathcal{G}_\sigma(p-q). \tag{44}$$

At the coincidence $p' = p$, with $\mathcal{G}_\sigma(0) = (2\pi\sigma^2)^{-1/2}$, this yields

$$K(p,q,t) = \kappa\,\bar{\psi}(p,t)\,\mathcal{G}_\sigma(p-q), \qquad \kappa = \frac{6\gamma}{\sqrt{2\pi\sigma^2}}. \tag{45}$$

The coupling tensor separates into a local amplitude $\kappa\bar{\psi}(p)$ and a spatial reach $\mathcal{G}_\sigma(p-q)$ that decays as a Gaussian with the same width σ that defines the coarse-graining resolution. The self-coupling $K(p,p) = 6\gamma\bar{\psi}(p)/(2\pi\sigma^2)$ scales as $1/\sigma^2$, establishing the baseline against which brake scaling is measured below. The product of two Gaussian kernels narrows to

$$\mathcal{G}_\sigma(p-q)^2 = \Gamma_0\,\mathcal{G}_{\sigma/\sqrt{2}}(p-q), \qquad \Gamma_0 = \mathcal{G}_\sigma(0) = (2\pi\sigma^2)^{-1/2}, \tag{46}$$

in one dimension. The recursive stress-energy tensor $T^{\mathrm{rec}}(p)$ therefore integrates over a neighborhood of width $\sigma/\sqrt{2}$, resolving structure at finer grain than the coarse-graining that produced it **(R2)**.

A.1.3 Brake Variation and Locality

Substituting the factored K into the brake functional $\mathcal{B}[K]$ and applying the kernel-narrowing identity gives

$$\mathcal{B} = \frac{\kappa^2 \Gamma_0}{2} \iint \bar{\psi}(p)^2 \, \mathcal{G}_{\sigma/\sqrt{2}}(p-q) \, \sqrt{g(p)}\sqrt{g(q)} \, dp \, dq. \tag{47}$$

Because K is linear in $\bar{\psi}$, the variation of $\mathcal{B}$ with respect to $\bar{\psi}$ is direct:

$$\frac{\delta \mathcal{B}}{\delta \bar{\psi}(r)} = \kappa^2 \Gamma_0 \, \bar{\psi}(r) \, g(r), \tag{48}$$

where the narrow Gaussian $\mathcal{G}_{\sigma/\sqrt{2}}$ has been evaluated at leading order. Computing $\delta\mathcal{B}/\delta C(r)$ requires threading the variation backward through the full dependency chain, $C \to \psi \to \bar{\psi} \to K \to \mathcal{B}$, which passes through a deconvolution of the Gaussian smearing ($\hat{\mathcal{G}}_\sigma^{-1}$) and a reconvolution through $\delta\mathcal{B}/\delta\psi$ ($\hat{\mathcal{G}}_\sigma$). For the cubic nonlinearity, these two operations cancel exactly:

$$\hat{\mathcal{G}}_\sigma^{-1}\left[\kappa^2 \Gamma_0 \, \hat{\mathcal{G}}_\sigma(\bar{\psi} \, g)\right] = \kappa^2 \Gamma_0 \, \bar{\psi} \, g. \tag{49}$$

The cancellation occurs because the cubic makes K linear in $\bar{\psi}$, so the second link in the chain ($\delta K/\delta\psi$) is independent of $\bar{\psi}$. The surviving non-trivial factor is the Jacobian of the reconstruction map, $h'(C) = 1/(1 + 3\gamma\bar{\psi}^2)$, entering through the chain rule at the $\psi \to C$ link. The brake variation collapses to a purely local expression **(R3)**:

$$\frac{\delta \mathcal{B}}{\delta C(r)} = \frac{\zeta \, \bar{\psi}(r) \, g(r)}{1 + 3\gamma\bar{\psi}(r)^2}, \qquad \zeta = \frac{9\gamma^2}{\pi^{3/2}\sigma^3}. \tag{50}$$

Higher-order nonlinearities (quintic, etc.) would break this cancellation and produce nonlocal brake terms.

A.1.4 Brake Saturation Dynamics

The denominator $1 + 3\gamma\bar{\psi}^2$ imposes a finite ceiling on regulatory capacity. For small fields ($\gamma\bar{\psi}^2 \ll 1$), the brake acts as a linear restoring force proportional to $\bar{\psi}$. For large fields ($\gamma\bar{\psi}^2 \gg 1$), the brake weakens as $1/|\bar{\psi}|$. Maximum braking occurs at $|\bar{\psi}| = 1/\sqrt{3\gamma}$; systems that exceed this magnitude have overwhelmed their own regulatory mechanism **(R4)**. The brake prefactor $\zeta \sim \gamma^2/\sigma^3$ grows faster than self-coupling ($\sim 1/\sigma^2$) as resolution sharpens, so the brake-to-coupling ratio scales as $1/\sigma$. Fine-resolution systems have disproportionately strong regulatory capacity relative to their coupling strength **(R5)**.

A.1.5 Reconstruction and Effective-Level Closure

The cubic $C = \bar{\psi} + \gamma\bar{\psi}^3$ is strictly monotonic for $\gamma > 0$ (its derivative $1 + 3\gamma\bar{\psi}^2$ is everywhere positive), so it admits a unique smooth inverse $\bar{\psi} = h(C)$ for all $C \in \mathbb{R}$. This map absorbs every reference to the sub-resolution field: the coupling tensor, brake variation, stress-energy, and all derived quantities become expressible purely in terms of C, g, and the seven parameters. The system closes at the effective level without requiring access to ψ at any point in its evolution (Axiom 2).

A.1.6 The Coupled Evolution System

The attractor potential takes the simplest binding form, $V(C) = \frac{\mu^2}{2}C^2$, with restoring force proportional to displacement (the generic leading-order behavior noted in the Axiom 6 remark). The autopoietic potential $\Phi(C) = \alpha_\Phi C^4$ is the lowest-order monomial satisfying $\beta_\Phi > 2$. This pair is the minimal choice producing a nontrivial effective potential with competing equilibria. The derivative $V'_{\text{eff}} = \mu^2 C - 4\alpha_\Phi C^3$ has the structure of an Allen–Cahn reaction term. The origin $C = 0$ is unstable (perturbations grow as $e^{\mu^2 t}$), and the two minima $C_* = \pm\mu/(2\sqrt{\alpha_\Phi})$ are stable (perturbations decay as $e^{-2\mu^2 t}$). The system cannot remain at zero coherence; non-trivial structure emerges spontaneously from arbitrarily small perturbations, and the choice of attractor basin breaks the $C \to -C$ symmetry. Autopoiesis is spontaneous symmetry breaking, structurally identical to the Higgs mechanism **(R6)**.

The attractor stability field,

$$A(C) = \frac{12\alpha_\Phi C^2 - \mu^2}{2\mu^2}, \tag{51}$$

derived directly from V''_{eff} and adding no new parameters, partitions the coherence field into attractor basins ($A > 0$, near $\pm C_*$) and the inter-basin region ($A < 0$, near the origin), with a sharp geometric boundary at $|C| = C_*/\sqrt{3}$. This field enters the MFE through the constraint-density contraction term $-2\eta_g A(C)(\partial_x C)^2$, whose sign reversal at the basin boundary means that gradients *contract* the metric inside attractor basins (semantic gravity) and *dilate* it between basins (geometric repulsion of unstable configurations). The semantic mass $M = D \cdot \rho \cdot A$ reduces from three ostensibly independent measurements to two: A is already encoded in the CFE's potential structure **(R7)**.

The dynamical CFE is

$$\partial_t C = \frac{1}{g}\left[\partial_{xx} C - \frac{(\partial_x g)(\partial_x C)}{2g}\right] + \mu^2 C - 4\alpha_\Phi C^3 + \frac{\lambda_B \zeta\, h(C)\, g}{1 + 3\gamma\, h(C)^2}. \tag{52}$$

The MFE follows from §3.3.4's general structure $\partial_t g_{\mu\nu} = -2R_{\mu\nu} + F_{\mu\nu}(D, K, A)$. In one spatial dimension, $R_{\mu\nu} \equiv 0$, so the Ricci flow term is absent and the MFE reduces to pure forcing. The forcing decomposes into two channels: constraint-density contraction, where the local gradient energy $(\partial_x C)^2$ modulated by attractor stability $A(C)$ drives metric response (contraction inside basins where $A > 0$, dilation between them where $A < 0$), and reflexive-density expansion, where the self-coupling amplitude $h(C)$ drives metric growth proportional to the existing metric. The coefficients η_g and ξ_g set the relative strength of each channel:

$$\partial_t g = -2\eta_g\, A(C)\, (\partial_x C)^2 + \frac{6\xi_g \gamma}{\pi\sigma^2}\, h(C)\, g. \tag{53}$$

Two state variables, two evolution equations, seven free parameters, all terms explicit.

A.1.7 The Dimensional Failure of the RFE

The Riemann tensor of any 1D Riemannian manifold vanishes identically: a single tangent direction provides no plane in which curvature can be defined. The Einstein tensor inherits this, $G_{11} = R_{11} - \frac{1}{2} g\, R = 0$ for every 1D metric. The tensorial RFE $G_{\mu\nu} = 8\pi G_s T^{\text{rec}}_{\mu\nu}$ reduces to $0 = 8\pi G_s T^{\text{rec}}_{11}$, and even the scalar reduction that rescues the 2+1 case (§A.2) has no content, since all curvature scalars vanish identically. The geometric coupling principle cannot operate; recursive stress-energy is well-defined, but the manifold provides no curvature degree of freedom for it to source. The metric evolves through the MFE's forcing alone. The absence of geometric constraint is the root cause of the metric blowup identified below.

A.1.8 Closure Conditions

Six conditions are tested. Local well-posedness follows from standard parabolic PDE theory (Evans, 2010): the CFE is semilinear parabolic, the MFE a pointwise ODE in g given C. The coherence field remains globally bounded by the Allen–Cahn maximum principle. The reconstruction map $h(C)$ is globally well-defined. All derived quantities (coupling tensor, brake, stress-energy, constraint density) are smooth and bounded wherever $g > 0$. The system closes in (C, g) without sub-resolution access (P4 self-consistency).

The one failure is in the metric's global behavior. In regions where $C \approx \pm C_*$ (stable equilibrium), spatial gradients vanish, the contraction term drops out, and the reflexive-density expansion drives g exponentially without bound. The root cause is the vanishing of intrinsic curvature in one spatial dimension: $R_{\mu\nu} \equiv 0$ on a 1D manifold, leaving the MFE no Ricci flow term to provide geometric self-damping. A second potential failure, metric collapse at domain walls between competing attractor basins, is independently resolved by $A(C)$: at the wall center ($C \approx 0$, $A < 0$), the contraction term reverses sign and becomes expansive, stabilizing the domain boundary without additional regulatory structure.

The CFE closes. The metric sector requires intrinsic curvature, which activates at $n \geq 2$.

A.1.9 Spatial Heterogeneity and Jensen's Correction

Integrating the CFE's Laplace–Beltrami term over the spatial domain does not yield zero when the coherence field is spatially non-uniform. Integration by parts on the diffusion contribution produces a remainder proportional to the spatial variance of $\bar{\psi}$ weighted by the local nonlinearity:

$$I_{\text{diff}} = \int h(C)\,[h'(C)]^3\,\frac{(\partial_x C)^2}{\sqrt{g}}\,dx > 0, \tag{54}$$

wherever $h(C) > 0$ (within a positive-coherence attractor basin). The positivity follows from Jensen's inequality (Jensen, 1906) applied to the concave relationship between C and the reflexive density D: smoothing a concave function over a spatially heterogeneous domain increases its spatial mean. A system with the same total coherence distributed unevenly across space generates more recursive depth than one with uniform coherence at the same total magnitude. The bonus is basin-dependent: where $h(C) < 0$ (near the inter-basin boundary at $|C| = C_*/\sqrt{3}$), the concavity reverses and the effect flips sign, confining productive heterogeneity to within-basin dynamics. The mechanism is purely geometric and invisible to any spatially homogeneous description **(R8)**.

A.1.10 Dimensional Stability of the Brake Functional

The brake variation (R3), saturation dynamics (R4), and resolution scaling (R5) are established on a single spatial slice. Whether the Hilbert–Schmidt normalization of $\mathcal{B}[K]$ remains well-defined when the spatial domain extends from one to n dimensions is a separate question, answerable through the perturbative structure of the functional itself.

A natural decomposition treats the n-dimensional spatial domain as a foliation of $(n-1)$-dimensional slices, each carrying its own reduced brake functional from the intra-slice coupling. Inter-slice coupling, governed by a transverse coupling parameter $\lambda_\perp$, introduces contributions from pairs of slices at increasing separation. The full brake functional admits a perturbative series (Legarde, personal communication):

$$\mathcal{B}_{n+1}[K] = \sum_{m=0}^{\infty} \lambda_\perp^m \left(\sum_{|i-j|=m} \mathcal{B}_{i,j}[K] \right), \tag{55}$$

where $\mathcal{B}_{i,j}[K]$ is the brake contribution from coupling between slices i and j at transverse separation m.

For the Gaussian kernel $\mathcal{G}_\sigma$, the inter-slice interaction strength inherits the kernel's squared decay through $\mathcal{B}$'s quadratic structure in K. The kernel-narrowing identity (§A.1.2) concentrates the coupling into the narrowed width $\sigma/\sqrt{2}$, so the inter-slice contribution decays super-exponentially:

$$B_m \sim B_0\,\exp\!\left(-m^2/\sigma^2\right). \tag{56}$$

The ratio test yields

$$L = \lambda_\perp \lim_{m\to\infty} \frac{B_{m+1}}{B_m} = \lambda_\perp \lim_{m\to\infty} \exp\!\left(-(2m+1)/\sigma^2\right) = 0, \tag{57}$$

establishing absolute convergence for any finite transverse coupling $\lambda_\perp$ and any positive resolution scale σ. The super-exponential decay in m^2 is a direct consequence of the Gaussian kernel's L^2 integrability, the same property that guarantees the Hilbert–Schmidt condition in the first place.

The convergence of the series establishes that the Hilbert–Schmidt normalization is dimensionally stable: regulatory capacity defined on lower-dimensional slices extends to the full manifold without divergence.

The series connects to the recurgence stability ratio (§3.2) through the brake saturation threshold of §A.1.4. As coupling intensity within a slice approaches $|\bar{\psi}| = 1/\sqrt{3\gamma}$, the effective contributions B_m grow relative to the geometric suppression from the Gaussian decay, and the effective ratio L rises toward unity. At $L \geq 1$, the series diverges: the brake functional can no longer be evaluated as a convergent sum, and autopoietic drive $\Phi(C)$ is no longer bounded by regulatory cost $\lambda_B\, \mathcal{B}[K]$. The transition at $L = 1$ is the perturbative signature of the $S_R > 1$ regime, the same bifurcation viewed through the dimensional extension's convergence structure.

At $n = 3$, the vanishing of the Weyl tensor (§A.3.3) ensures conformal flatness, so the series captures total curvature content with no residual propagating degrees of freedom. The foliation series and geometric self-transparency form complementary results, the former establishing that the brake converges across slices, the latter that nothing escapes the slices' collective accounting.

1+1 Structural Results.

R1. Linear-$\mathcal{F}$ impossibility: any linear coarse-graining gives $K \equiv 0$.

R2. Stress-energy resolution enhancement: T^{rec} resolves at $\sigma/\sqrt{2}$, finer than the coarse-graining scale.

R3. Brake cancellation to locality: convolution–deconvolution cancellation collapses $\delta\mathcal{B}/\delta C$ to a pointwise expression.

R4. Brake saturation: maximum regulatory capacity at $|\bar{\psi}| = 1/\sqrt{3\gamma}$, with failure beyond.

R5. Resolution scaling: brake-to-coupling ratio $\sim 1/\sigma$; fine-resolution systems self-regulate more strongly.

R6. Autopoiesis as SSB: double-well effective potential producing spontaneous symmetry breaking at $\pm C_*$.

R7. Derived $A(C)$: attractor stability encoded in potential structure, reducing $M = D \cdot \rho \cdot A$ to two independent fields.

R8. Jensen's diffusion bonus: spatial heterogeneity increases total reflexive density through the concavity of $D(C)$.

Dimensional stability (§A.1.10, Legarde): The perturbative series convergence extends R3–R5 to arbitrary spatial dimension.
All eight structural results, derived on a single spatial slice, compose across arbitrary foliation without loss of convergence.

A.2 The 2+1 Model

Two spatial dimensions, one time parameter. The minimal extension where intrinsic curvature is nontrivial and the causal loop closes.

The 1+1 model's metric blowup traced to a dimensional identity: $R_{\mu\nu} \equiv 0$ on any 1D Riemannian manifold. Two spatial dimensions is the minimal setting where intrinsic curvature becomes dynamical, the geometric coupling principle produces a constraint, and the full causal loop $C \to T^{\mathrm{rec}}_{\mu\nu} \to K \to g_{\mu\nu} \to C$ closes.

A.2.1 The Scalar Reduction of the Coupling Principle

At $n = 2$, the Riemann tensor has one independent component, the Gaussian curvature $\mathcal{K}$. The Ricci tensor becomes $R_{\mu\nu} = \mathcal{K}\, g_{\mu\nu}$, and the scalar curvature is $R = 2\mathcal{K}$. The Einstein tensor,

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = \mathcal{K} g_{\mu\nu} - \mathcal{K} g_{\mu\nu} = 0, \tag{58}$$

vanishes identically, a dimensional identity shared with 2+1 gravity in general relativity (Deser et al., 1984). The tensorial RFE would force $T^{\mathrm{rec}}_{\mu\nu} = 0$, which is trivial. The geometric coupling principle must be expressed in its dimensionally appropriate scalar form:

$$\mathcal{K} = 4\pi G_s\, g^{\mu\nu} T^{\mathrm{rec}}_{\mu\nu}. \tag{59}$$

This is the natural 2D analog, structurally parallel to Jackiw–Teitelboim gravity (Teitelboim, 1983; Jackiw, 1985). The equation is non-trivial, constraining the Gaussian curvature at every point to equal the local trace of recursive stress-energy.

A.2.2 Conformal Parameterization and Isothermal Coordinates

Any 2D Riemannian metric admits isothermal coordinates $g_{\mu\nu} = e^{2\varphi}\delta_{\mu\nu}$, where the conformal factor $\varphi(x, y, t)$ encodes the entire metric (Chern, 1955). This is exact, not an approximation. All geometric quantities derive from φ. The Gaussian curvature is

$$\mathcal{K} = -e^{-2\varphi}\,\Delta\varphi. \tag{60}$$

The Laplace–Beltrami operator reduces from the general form

$$\Delta_g f = \frac{1}{\sqrt{g}}\,\partial_\mu(\sqrt{g}\,g^{\mu\nu}\partial_\nu f) \tag{61}$$

to the simplified form

$$\Delta_g f = e^{-2\varphi}\,\Delta f, \tag{62}$$

where the $e^{2\varphi}$ and $e^{-2\varphi}$ factors cancel in $\sqrt{g}\cdot g^{\mu\nu}$. The volume element is $\sqrt{g} = e^{2\varphi}$. The scalar RFE becomes a Poisson equation for the conformal factor:

$$\Delta\varphi = -4\pi G_s\,\mathrm{Tr}_{\mathrm{flat}}(T^{\mathrm{rec}})\,. \tag{63}$$

Positive stress-energy trace produces concave φ (positive curvature). The Green's function is the 2D logarithmic potential: mass at one point curves geometry everywhere, with influence decaying as $\log r$.

A.2.3 Generalization of the Matter Sector

The coarse-graining functional, coupling tensor, and brake variation carry over from §A.1 with modified prefactors. The 2D Gaussian kernel

$$\mathcal{G}_\sigma(\mathbf{r}) = (2\pi\sigma^2)^{-1}\exp\left(-\frac{|\mathbf{r}|^2}{2\sigma^2}\right) \tag{64}$$

replaces its 1D counterpart, and the recursive coupling tensor retains the same factored form:

$$K(\mathbf{p},\mathbf{q}) = \kappa\,\bar{\psi}(\mathbf{p})\,\mathcal{G}_\sigma(\mathbf{p}-\mathbf{q}), \qquad \kappa = \frac{3\gamma}{\pi\sigma^2}. \tag{65}$$

The brake cancellation to locality depends on the convolution–deconvolution structure of the Gaussian and the linearity of K in $\bar{\psi}$, both dimension-independent. The result is the same local brake variation with the exact 2D prefactor

$$\zeta = \frac{9\gamma^2}{4\pi^3\sigma^6}, \tag{66}$$

(versus $9\gamma^2/(\pi^{3/2}\sigma^3)$ in 1D; both obey the general scaling γ^2/σ^{3d}). Brake saturation at $|\bar{\psi}| = 1/\sqrt{3\gamma}$ is a pointwise algebraic identity, unchanged. The reconstruction map $h(C)$ and attractor stability field $A(C)$ are also pointwise constructions that transfer without modification. The kernel-narrowing identity $\mathcal{G}_\sigma(\mathbf{r})^2 = \Gamma_0\,\mathcal{G}_{\sigma/\sqrt{2}}(\mathbf{r})$ is dimension-independent, so the stress-energy resolution enhancement by $\sqrt{2}$ generalizes.

All eight structural results from §A.1 survive. Five depend only on the algebraic structure of $\mathcal{F}$ and transfer without modification (R1–R4, R7). Resolution scaling (R5) steepens: the brake-to-coupling ratio scales as $1/\sigma^2$ in 2D versus $1/\sigma$ in 1D. The remaining two gain new content. Autopoiesis as SSB (R6) acquires curvature-modulated spatial texture (§A.2.11). Jensen's diffusion bonus (R8) amplifies through angular modes: the 2D spatial variance receives contributions from all directions ($\mathrm{Var}_{\mathrm{2D}} \geq \mathrm{Var}_{\mathrm{1D}}|_{\text{any direction}}$), with circular harmonics ($m \geq 1$) contributing gradient energy proportional to m^2/r^2 that has no 1D analog. The 2D bonus is at least as large as the 1D bonus.

A.2.4 Dimension-Dependent Resolution Scaling

The hierarchy of resolution-dependent quantities obeys a general d-dimensional pattern. Self-coupling scales as $1/\sigma^{2d}$, the brake prefactor as $1/\sigma^{3d}$, and the brake-to-coupling ratio as $1/\sigma^{d}$. In 2D: self-coupling $\sim 1/\sigma^4$, brake prefactor $\sim 1/\sigma^6$, brake-to-coupling ratio $\sim 1/\sigma^2$. Each additional spatial dimension steepens the regulatory advantage by one power of $1/\sigma$. This is the first dimension-dependent quantitative prediction produced by the toy model program.

A.2.5 The Recursive Stress-Energy Tensor and Kinetic Trace Cancellation

The recursive stress energy tensor is

$$T^{\mathrm{rec}}_{\mu\nu} = \partial_\mu C\,\partial_\nu C - g_{\mu\nu}\,\mathcal{L}, \tag{67}$$

where

$$\mathcal{L} = \tfrac{1}{2} g^{\alpha\beta}\partial_\alpha C\,\partial_\beta C - V(C) + \Phi(C) - \lambda_B \mathcal{B}. \tag{68}$$

In conformal gauge, the three independent components are:

$$T^{\mathrm{rec}}_{xx} = (\partial_x C)^2 - e^{2\varphi}\mathcal{L}, \qquad T^{\mathrm{rec}}_{yy} = (\partial_y C)^2 - e^{2\varphi}\mathcal{L}, \qquad T^{\mathrm{rec}}_{xy} = (\partial_x C)(\partial_y C). \tag{69}$$

The diagonal components carry the Lagrangian subtraction; the off-diagonal component is a pure gradient product measuring shear stress of coherence flow. The curved trace is

$$g^{\mu\nu} T^{\mathrm{rec}}_{\mu\nu} = 2\mathcal{E}_k - n\mathcal{L}, \tag{70}$$

where $\mathcal{E}_k = \frac{1}{2} g^{\mu\nu}\partial_\mu C\,\partial_\nu C$ is the kinetic energy density and n counts spatial dimensions. Expanding $\mathcal{L} = \mathcal{E}_k - V + \Phi - \lambda_B \mathcal{B}$ gives

$$g^{\mu\nu} T^{\mathrm{rec}}_{\mu\nu} = (2-n)\mathcal{E}_k + n\big(V - \Phi + \lambda_B \mathcal{B}\big). \tag{71}$$

At exactly $n = 2$, the coefficient $(2-n)$ vanishes and the kinetic term drops out:

$$g^{\mu\nu} T^{\mathrm{rec}}_{\mu\nu}\big|_{n=2} = 2\big(V(C) - \Phi(C) + \lambda_B \mathcal{B}\big). \tag{72}$$

This algebraic identity holds for any scalar Lagrangian with standard kinetic term in two spatial dimensions, as a consequence of the conformal invariance of the scalar kinetic term in 2D **(R9)**. At any other dimension, kinetic energy contributes to the trace with coefficient $(2-n)$ and therefore to curvature sourcing.

A.2.6 Curvature–Potential Correspondence

Substituting the trace $g^{\mu\nu} T^{\mathrm{rec}}_{\mu\nu} = 2\big(V - \Phi + \lambda_B \mathcal{B}\big)$ into the scalar RFE (59) absorbs the factor of 2 into the coupling constant, yielding a three-term algebraic relationship between Gaussian curvature and the local potential balance:

$$\mathcal{K} = 8\pi G_s\big(V(C) - \Phi(C) + \lambda_B \mathcal{B}\big). \tag{73}$$

Attractor binding V and regulatory cost $\lambda_B \mathcal{B}$ source positive curvature (geometry contracts locally). Autopoietic drive Φ sources negative curvature (geometry expands locally). The direction and speed of coherence flow do not affect curvature; manifold geometry is determined by the dispositional structure of the field configuration alone. This decoupling of structure from dynamics is specific to $n = 2$ **(R10)**.

A.2.7 The Gauss–Bonnet Balance Law

On a closed 2D manifold with Euler characteristic χ, the Gauss–Bonnet theorem (do Carmo, 1992) requires $\int \mathcal{K}\,dA = 2\pi\chi$. On a torus ($\chi = 0$, the natural choice for periodic boundary conditions), integrating (73) over the manifold gives

$$\int_{\mathcal{M}} \big(V(C) + \lambda_B \mathcal{B}\big)\,dA = \int_{\mathcal{M}} \Phi(C)\,dA. \tag{74}$$

Total attractor binding plus total regulatory cost equals total autopoietic drive. This is a topological constraint on the field-theoretic content, enforced by the Gauss–Bonnet theorem with no adjustable parameters. Three consequences immediately follow. A manifold cannot be everywhere attractor-dominated: $V > \Phi$ everywhere would force $\int \mathcal{K}\, dA > 0$, violating the constraint. A manifold cannot be everywhere autopoietically dominated, by the symmetric argument. Every attractor basin (positive curvature) is necessarily offset by a generative region (negative curvature) of equal total curvature. Stability and generativity are complementary on the torus; neither can dominate globally **(R11)**.

A.2.8 Curvature–Diffusion Feedback

The Laplace–Beltrami operator $\Delta_g = e^{-2\varphi}\Delta$ couples the scalar RFE back into the CFE, creating a feedback loop absent in 1+1. In positive-curvature regions (attractor-dominated, $V > \Phi$), the conformal factor is locally maximal, the metric is expanded, and Δ_g is reduced: diffusion slows and coherence is more tightly held. In negative-curvature regions (autopoietically dominated, $\Phi > V$), the conformal factor is locally minimal, the metric is contracted, and Δ_g is enhanced: diffusion accelerates and coherence flows more freely. The geometry actively reinforces whatever local character the field configuration already has. Attractor basins self-stabilize through curvature-mediated diffusion suppression; generative regions self-sustain through curvature-mediated diffusion enhancement. A linearized estimate quantifies the strength: for a coherence feature of spatial scale L in a region of Gaussian curvature $\mathcal{K}$, the Poisson equation gives a conformal perturbation $\delta\varphi \sim \mathcal{K}L^2$, and the effective diffusion coefficient $e^{-2\varphi}$ shifts by a fraction $\sim 2|\mathcal{K}|L^2$. The dimensionless product $|\mathcal{K}|L^2$ controls the regime: for $|\mathcal{K}|L^2 \ll 1$ (curvature radius much larger than feature size), the self-reinforcement is perturbative and the flat-space results of §A.1 remain accurate; for $|\mathcal{K}|L^2 \sim O(1)$, the geometric feedback is order-unity and reshapes the dynamics **(R12)**.

A.2.9 Topological Regulation of Local Feedback

The curvature–diffusion feedback is locally unbounded but globally limited by the Gauss–Bonnet constraint. An attractor basin that deepens (increasing local positive curvature) forces compensating negative curvature elsewhere, which accelerates diffusion in the compensating region, producing generative coherence that may eventually feed back into the basin. The combination of local self-reinforcement and global topological limitation constitutes a geometric stability mechanism with no free parameters. The mechanism predicts a timescale separation: local entrenchment is fast (set by the curvature–diffusion feedback rate), while global rebalancing is slow (set by diffusion transport across the manifold) **(R13)**.

A.2.10 The Geometric Inflation Pathway

The curvature–diffusion feedback interacts with the brake saturation threshold from §A.1 to create a geometric pathway from stability to regulatory failure. In positive-curvature regions, the expanded metric slows diffusion, which accumulates coupling density ($|\bar{\psi}|$ increases locally). If $|\bar{\psi}|$ crosses $1/\sqrt{3\gamma}$, the brake saturates and regulatory cost begins decreasing with further coupling intensity. The very curvature that stabilizes an attractor basin also drives the basin toward the inflation boundary. The question is whether global topological rebalancing redistributes curvature before local accumulation crosses the threshold, a competition between two timescales (τ_{local} for coupling accumulation, τ_{global} for Gauss–Bonnet rebalancing) that defines a structural phase boundary in the theory's parameter space **(R14)**.

A.2.11 Curvature-Modulated Symmetry Breaking

The SSB equilibria $C_* = \pm\mu/(2\sqrt{\alpha_\Phi})$ from §A.1 acquire spatial texture on the curved manifold. The CFE in conformal gauge is

$$\Delta C = e^{2\varphi}\left[V'_{\text{eff}}(C) + \lambda_B\, \delta\mathcal{B}/\delta C\right], \tag{75}$$

where the conformal factor $e^{2\varphi}$ modulates the effective forcing at every point. The equilibrium values are unchanged, but their stability is position-dependent: the restoring force toward C_* is amplified in positive-curvature regions (large $e^{2\varphi}$) and weakened in negative-curvature regions (small $e^{2\varphi}$). The system breaks symmetry non-uniformly, with the depth of the broken-symmetry state varying across the manifold according to the local curvature **(R15)**.

A.2.12 Closure of the Causal Loop

With all terms assembled, the 2+1 model produces a coupled system of nonlinear elliptic PDEs for $C(x, y)$ and $\varphi(x, y)$. The CFE determines coherence given geometry:

$$\Delta C = e^{2\varphi}\left[\mu^2 C - 4\alpha_\Phi C^3 + \frac{\lambda_B \zeta\, h(C)\, e^{2\varphi}}{1 + 3\gamma\, h(C)^2}\right]. \tag{76}$$

(The outer $e^{2\varphi}$ inverts the Laplace–Beltrami; the inner $e^{2\varphi}$ in the brake numerator is the volume factor $\sqrt{g}$ from the brake variation defined in §A.1.)

The scalar RFE determines geometry given coherence:

$$\Delta\varphi = -8\pi G_s\, e^{2\varphi}\big(V(C) - \Phi(C) + \lambda_B \mathcal{B}\big). \tag{77}$$

A self-consistent solution is a fixed point of the loop. The Gauss–Bonnet constraint provides a necessary integrability condition for periodic solutions on the torus (the Poisson source must integrate to zero; existence then follows from standard elliptic regularity). The MFE governs how the system approaches equilibrium. As in §A.1, the forcing decomposes into two explicit channels: attractor-weighted contraction, where the gradient energy $|\nabla C|^2$ modulated by $A(C)$ drives metric response, and reflexive-density expansion, where the self-coupling amplitude $h(C)$ drives metric growth:

$$\partial_t \varphi = e^{-2\varphi}\Delta\varphi - \eta_g\, A(C)\, e^{-2\varphi}|\nabla C|^2 + \frac{\xi_g \kappa}{2}\, h(C). \tag{78}$$

The contraction channel contracts the metric inside attractor basins ($A > 0$) and dilates it between basins ($A < 0$); the expansion channel drives metric growth wherever the sub-resolution field $h(C)$ is nonzero. The unforced sector ($\eta_g = \xi_g = 0$) is the normalized Ricci flow on surfaces, whose global regularity on closed manifolds is well established (Hamilton, 1988): the flow smooths curvature, shrinks positive-curvature regions, and drives metrics toward constant-curvature configurations. The forcing term resists this homogenization, injecting curvature where recursive coupling is intense. If the forcing has a traceless component (anisotropic coherence flow), the conformal gauge is not preserved under time evolution. The conformal parameterization is exact at each instant by the existence of isothermal coordinates, but the time evolution may require updating the conformal frame. For the minimal model, conformal forcing is adopted as the leading-order treatment. Near spatially homogeneous equilibria, coherence gradients are perturbatively small, the traceless stress $\hat{T}_{\mu\nu}$ is second-order in gradient amplitudes, and the conformal (trace) sector captures the dominant metric response.

A.2.13 Unconstrained Shear Stress and the Motivation for $n = 3$

The scalar RFE constrains the trace of $T^{\text{rec}}_{\mu\nu}$ but not its traceless part. The off-diagonal stress component $T^{\text{rec}}_{xy} = (\partial_x C)(\partial_y C)$ enters the MFE forcing directly but is not constrained by the curvature equation. Directional selectivity of coupling patterns shapes metric evolution without geometric constraint.

2+1 Structural Results.

R9. Kinetic trace cancellation: $(2-n)\mathcal{E}_k$ vanishes at $n=2$; curvature decouples from coherence dynamics (dimension-specific, breaks at $n=3$).

R10. Curvature–potential correspondence: $\mathcal{K} = 8\pi G_s\big(V - \Phi + \lambda_B\mathcal{B}\big)$; manifold geometry determined by dispositional structure alone.

R11. Gauss–Bonnet balance law: total attractor binding plus regulatory cost equals total autopoietic drive on the torus; topological constraint with no free parameters.

R12. Curvature–diffusion self-reinforcement: positive curvature slows diffusion (stabilizing basins), negative curvature accelerates it (sustaining generativity).

R13. Topological self-limitation: local self-reinforcement is globally bounded by the Gauss–Bonnet constraint; entrenchment and rebalancing compete on separated timescales.

R14. Geometric inflation pathway: curvature-mediated coupling accumulation drives $|\bar{\psi}|$ toward brake saturation threshold $1/\sqrt{3\gamma}$; overstability seeds its own regulatory failure.

R15. Non-uniform symmetry breaking: SSB equilibria acquire position-dependent stability modulated by the conformal factor $e^{2\varphi}$.

All eight 1+1 results (R1–R8) generalize: five carry without modification, one steepens (R5, resolution scaling), two gain qualitative content (R6, R8). R9 is the one informative break: it holds at exactly $n=2$ and fails at $n=3$.

A.3 The 3+1 Model

Three spatial dimensions, one time parameter. Curvature becomes directional, the Einstein tensor operates in its intended form, and the Weyl tensor's vanishing identifies three dimensions as the completeness checkpoint.

The 2+1 model closed the causal loop through a scalar equation relating Gaussian curvature to the stress-energy trace. Three structural constraints of two-dimensional geometry limited what that closure could test. The Einstein tensor vanished identically ($G_{\mu\nu} \equiv 0$ in 2D), forcing the coupling principle into a scalar reduction that constrained curvature magnitude while leaving shear stress geometrically unconstrained. The kinetic trace cancellation held exactly: the coefficient $(2-n)$ vanishes at $n=2$, decoupling active coherence flow from curvature sourcing. And conformal flatness was guaranteed by a theorem of 2D Riemannian geometry, making conformal gauge a complete parameterization rather than an approximation. At $n=3$, all three constraints lift.

A.3.1 The Conformal Parameterization and the Cotton Tensor

The conformal gauge $g_{\mu\nu} = e^{2\varphi}\delta_{\mu\nu}$ is no longer guaranteed by an existence theorem in three dimensions. It covers conformally flat geometries only, a proper subset of all 3D metrics. The obstruction to conformal flatness is measured by the Cotton tensor (Cotton, 1899),

$$\mathfrak{C}_{\mu\nu\lambda} = \nabla_\lambda\big(R_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}R\big) - \nabla_\nu\big(R_{\mu\lambda} - \tfrac{1}{4}g_{\mu\lambda}R\big)\,, \tag{79}$$

which serves the same diagnostic role in three dimensions the Weyl tensor serves at $n \geq 4$: a 3D metric is conformally flat if and only if $\mathfrak{C}_{\mu\nu\lambda} = 0$. Within the conformal parameterization the Cotton tensor vanishes by construction; its relevance is diagnostic, detecting when the metric develops conformal structure that the single-scalar parameterization cannot capture.

A.3.2 The Tensorial RFE

The Ricci tensor in conformal gauge is

$$R_{\mu\nu} = -(n-2)[\partial_\mu\partial_\nu\varphi - \partial_\mu\varphi\,\partial_\nu\varphi] - \left[\Delta\varphi + (n-2)|\nabla\varphi|^2\right]\delta_{\mu\nu}, \tag{80}$$

an independent symmetric tensor with six components at $n = 3$. The Hessian $\partial_\mu\partial_\nu\varphi$ and quadratic gradient $\partial_\mu\varphi\,\partial_\nu\varphi$ that distinguish it from the 2D case both carry the coefficient $(n-2)$. The scalar curvature

$$R = e^{-2\varphi}\left[-2(n-1)\Delta\varphi - (n-1)(n-2)|\nabla\varphi|^2\right] \tag{81}$$

picks up a gradient energy contribution $(n-1)(n-2)|\nabla\varphi|^2$ absent in two dimensions: regions where φ varies steeply are more curved than the Laplacian alone would predict. The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ simplifies at $n = 3$: the $|\nabla\varphi|^2\delta_{\mu\nu}$ contributions from $R_{\mu\nu}$ and $\frac{1}{2}g_{\mu\nu}R$ enter with equal magnitude and opposite sign, cancelling to leave

$$G_{\mu\nu} = -\partial_\mu\partial_\nu\varphi + \partial_\mu\varphi\,\partial_\nu\varphi + \Delta\varphi\,\delta_{\mu\nu}, \tag{82}$$

with six non-trivial independent components at $n = 3$. The tensorial RFE $G_{\mu\nu} = 8\pi G_s T^{\mathrm{rec}}_{\mu\nu}$ splits into two families. The three diagonal equations ($\mu = \nu = i$, with j, k the remaining indices) take the form:

$$\partial_j^2\varphi + \partial_k^2\varphi + (\partial_i\varphi)^2 = 8\pi G_s\,T^{\mathrm{rec}}_{ii}, \tag{83}$$

relating the conformal factor's curvature in the perpendicular plane and its squared gradient along the axis to the coupling pressure in each direction. The three off-diagonal equations ($\mu \neq \nu$) take the form:

$$-\partial_\mu\partial_\nu\varphi + \partial_\mu\varphi\,\partial_\nu\varphi = 8\pi G_s\,T^{\mathrm{rec}}_{\mu\nu}, \tag{84}$$

constraining the cross-derivatives of φ against the shear components of recursive stress-energy. In 2+1, the shear stress existed in the stress-energy tensor but no curvature equation constrained it; three dimensions closes this gap **(R18)**. The trace yields:

$$R = -16\pi G_s\,\mathrm{Tr}_g(T^{\mathrm{rec}})\,, \tag{85}$$

the scalar descendant of the 2+1 model's entire curvature constraint, now one component of six. The five traceless components distribute curvature directionally according to the directional distribution of coupling. The contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$, an identity of Riemannian geometry, yields covariant conservation $\nabla^\mu T^{\mathrm{rec}}_{\mu\nu} = 0$ through the Einstein equation. Covariant conservation of recursive stress-energy follows from the geometry with no additional postulate, providing three integrability conditions that constrain which coupling configurations are geometrically permissible and, through constraint-evolution compatibility, pin the MFE forcing (§A.3.12).

A.3.3 Geometric Self-Transparency

The Riemann tensor has $n^2(n^2-1)/12$ independent components, giving six at $n = 3$. The Ricci tensor has $n(n+1)/2$ components, also six at $n = 3$. The Weyl tensor, measuring the part of Riemann not determined by Ricci, has $n(n+1)(n+2)(n-3)/12$ components; the factor $(n-3)$ annihilates the expression at $n = 3$. The Weyl tensor vanishes identically on every three-dimensional Riemannian manifold, and the Riemann decomposition collapses to

$$\begin{aligned} R_{\mu\nu\rho\sigma} = g_{\mu\rho}R_{\nu\sigma} - g_{\mu\sigma}R_{\nu\rho} - g_{\nu\rho}R_{\mu\sigma} + g_{\nu\sigma}R_{\mu\rho} \\ - \frac{R}{2}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\,. \end{aligned} \tag{86}$$

Every component of the Riemann tensor is an algebraic function of $R_{\mu\nu}$, R, and $g_{\mu\nu}$ (Wald, 1984). All curvature on the 3D manifold is determined by local stress-energy through the Einstein equation. If recursive coupling vanishes at a point, the Ricci tensor vanishes, and since the Weyl tensor is identically zero, the Riemann tensor vanishes: the region is flat. No mechanism exists by which curvature at one location can warp the geometry of a region carrying zero local coupling density **(R16)**. At $n = 4$, the Weyl tensor acquires 10 independent components carrying curvature that propagates through regions of zero stress-energy, analogous to gravitational radiation in general relativity. Three dimensions is the last dimension where geometric self-transparency holds, and therefore the most constrained non-trivial setting for the covariant formalism: the Einstein tensor has full tensorial structure, the Bianchi identity provides covariant conservation, the Ricci flow runs in its genuinely three-dimensional form (Perelman, 2003), and every geometric distortion has a locally identifiable source. Extension to $n \geq 4$, where the Weyl tensor adds propagating degrees of freedom without changing the equation structure, follows from the covariance of the formalism; the Weyl tensor populates an existing slot in the Riemann decomposition (§A.4.2).

A.3.4 The $(n-2)$ Organizing Coefficient

The entire dimensional transition from 2+1 to 3+1 is organized by a single coefficient. Three effects absent in two dimensions activate simultaneously at $n = 3$, all governed by the factor $(n-2)$; a fourth, the variational absorption of the interpretive operator, follows as their consequence. The Laplace–Beltrami operator acquires a first-order advection term:

$$\Delta_g f = e^{-2\varphi}(\Delta f + (n-2)\,\nabla\varphi \cdot \nabla f)\,. \tag{87}$$

At $n = 2$, the operator reduces to a conformal rescaling of the flat Laplacian. At $n = 3$, the conformal gradient $\nabla\varphi$ imposes a drift field that steers coherence toward regions of expanded metric and away from regions of contracted metric. The Ricci tensor's off-diagonal and traceless components, carrying the directional curvature structure absent in two dimensions, enter with the same coefficient. The stress-energy trace coefficient $(2-n)$, whose vanishing at $n = 2$ produced the kinetic trace cancellation of §A.2.5, becomes -1 at $n = 3$: kinetic energy re-enters curvature sourcing, and active coherence flow curves the manifold. Conformal advection, anisotropic Ricci curvature, and kinetic-curvature coupling all activate through this single coefficient **(R23)**.

A.3.5 Steepened Scaling of the Matter Sector

The coupling tensor, brake variation, and stress-energy tensor built for the 1+1 and 2+1 models extend to three spatial dimensions through the 3D Gaussian smearing kernel

$$\mathcal{G}_\sigma(\mathbf{r}) = (2\pi\sigma^2)^{-3/2} \exp\left(-\frac{|\mathbf{r}|^2}{2\sigma^2}\right). \tag{88}$$

The factored structure of the recursive coupling tensor persists:

$$K(\mathbf{p},\mathbf{q}) = \kappa\,\bar{\psi}(\mathbf{p})\,\mathcal{G}_\sigma(\mathbf{p}-\mathbf{q}), \qquad \kappa = \frac{6\gamma}{(2\pi\sigma^2)^{3/2}}. \tag{89}$$

The brake cancellation to locality, which depends on the convolution–deconvolution structure of the Gaussian and the linearity of K in $\bar{\psi}$, is an algebraic property of convolution operators on $\mathbb{R}^d$ and holds in any dimension. The brake variation remains:

$$\frac{\delta\mathcal{B}}{\delta C(r)} = \frac{\zeta\,\bar{\psi}(r)\,g(r)}{1 + 3\gamma\bar{\psi}(r)^2}, \qquad \zeta \sim \frac{\gamma^2}{\sigma^9}. \tag{90}$$

The resolution scaling hierarchy obeys a general d-dimensional pattern: self-coupling scales as $1/\sigma^{2d}$, the brake prefactor as $1/\sigma^{3d}$, and the brake-to-coupling ratio as $1/\sigma^d$. At $d = 3$, halving the resolution scale σ multiplies self-coupling by $2^6 = 64$, the brake prefactor by $2^9 = 512$, and the brake-to-coupling ratio by $2^3 = 8$. Three data points at $d = 1, 2, 3$ confirm the general pattern, and the formula for arbitrary d follows from the Gaussian kernel's dimensional scaling without further assumptions. The kernel-narrowing identity $\mathcal{G}_\sigma(\mathbf{r})^2 = \Gamma_0\,\mathcal{G}_{\sigma/\sqrt{2}}(\mathbf{r})$ is dimension-independent, so the stress-energy resolution enhancement by $\sqrt{2}$ carries over.

A.3.6 Directional Structure of the Recursive Stress-Energy Tensor

The six independent components of $T^{\text{rec}}_{\mu\nu}$ in conformal gauge split into two families that feed the two families of Einstein equations. The three diagonal components $T^{\text{rec}}_{ii} = (\partial_i C)^2 - e^{2\varphi}\mathcal{L}$ measure normal stress along each axis: a direction along which the coherence field varies steeply carries more stress-energy than directions along which it is smooth. The three off-diagonal components $T^{\text{rec}}_{\mu\nu} = \partial_\mu C\,\partial_\nu C$ ($\mu \neq \nu$) are pure gradient products with no Lagrangian subtraction, measuring the shear stress of coherence flow between pairs of axes. The diagonal stresses source the diagonal Einstein equations, governing how the conformal factor curves in the plane perpendicular to each axis. The shear stresses source the off-diagonal Einstein equations, governing how the conformal factor twists between pairs of axes. In 2+1, shear stress existed in the recursive stress-energy tensor, but no curvature equation constrained it. In three dimensions, this structural gap is closed.

A.3.7 The Kinetic Trace Cancellation Breaks

The curved trace of the stress-energy tensor (§A.2.5) is

$$g^{\mu\nu}T^{\text{rec}}_{\mu\nu} = (2-n)\mathcal{E}_k + n\big(V - \Phi + \lambda_B\mathcal{B}\big), \tag{91}$$

where $\mathcal{E}_k = \frac{1}{2}g^{\mu\nu}\partial_\mu C\,\partial_\nu C$ is the kinetic energy density. At $n=2$, the coefficient $(2-n)$ vanished, and kinetic energy dropped completely from the curvature source. At $n=3$, the coefficient is -1:

$$g^{\mu\nu}T^{\text{rec}}_{\mu\nu}\big|_{n=3} = -\mathcal{E}_k + 3\big(V - \Phi + \lambda_B\mathcal{B}\big). \tag{92}$$

The trace of the Einstein equation yields

$$R = 16\pi G_s\,\mathcal{E}_k - 48\pi G_s\big(V - \Phi + \lambda_B\mathcal{B}\big). \tag{93}$$

Kinetic energy reduces scalar curvature. Active coherence flow counteracts the geometric stabilization that the potential balance builds **(R17)**. The kinetic trace cancellation (R9) was a zero-crossing of the $(2-n)$ coefficient, holding at exactly $n=2$ and nowhere else. It is the only result among R1–R15 that breaks at $n=3$; the remaining fourteen generalize directly or with enrichment (§A.4.1). The six-component view reveals the directional complement: the off-diagonal Einstein equations are sourced entirely by kinetic gradients ($G_{\mu\nu} = 8\pi G_s\,\partial_\mu C\,\partial_\nu C$ for $\mu \neq \nu$), so the directional curvature structure responds to the present state of motion of the coherence field alongside its configuration. The causal loop tightens: perturbations affect curvature immediately through their kinetic energy, before any structural change has developed.

A.3.8 Conformal Advection in the CFE

The coherence field equation in conformal gauge,

$$\partial_t C = e^{-2\varphi}(\Delta C + \nabla\varphi\cdot\nabla C) - V'(C) + \Phi'(C) - \lambda_B\,\frac{\delta\mathcal{B}}{\delta C}, \tag{94}$$

differs from the 2+1 CFE by one term: $e^{-2\varphi}\nabla\varphi\cdot\nabla C$. The 2+1 spatial operator $e^{-2\varphi}\Delta C$ was isotropic diffusion with a position-dependent rate; the conformal factor modulated how fast coherence diffused but imposed no directional preference. The 3+1 operator $e^{-2\varphi}(\Delta C + \nabla\varphi\cdot\nabla C)$ adds advection through the Laplace–Beltrami's $(n-2)$ coefficient. The conformal gradient $\nabla\varphi$ imposes a drift velocity on the coherence field, steering it toward regions of expanded metric. Near the center of a well-developed attractor basin, where φ is locally maximal, the advection term pulls coherence inward from the surrounding region. The geometry of the manifold actively concentrates coherence around existing attractor basins and depletes the surrounding regions. The advection converts the equilibrium CFE from a self-adjoint to a non-self-adjoint elliptic PDE, encoding the geometry's directional preference in the operator structure **(R20)**.

The advection interacts with the kinetic trace break to create a kinetic valve on the curvature–diffusion feedback. In 2+1, positive curvature slowed diffusion, trapped coherence maintained the stress-energy sourcing the curvature, and the feedback loop was self-reinforcing with no internal release mechanism. In 3+1, active coherence flow ($\mathcal{E}_k > 0$) reduces the scalar curvature below what the potentials alone would produce, which reduces the metric expansion, which increases the effective diffusion coefficient. The curvature–diffusion trap partially opens when coherence is in motion and reasserts when flow subsides. The curvature–potential correspondence of §A.2.6 survives as the $\mathcal{E}_k = 0$ limit.

A.3.9 Spherical Harmonic Enrichment of Jensen's Bonus

The Jensen correction from the nonlinear $\mathcal{F}$ is proportional to the spatial variance of $\bar{\psi}$. In 1+1, only radial modes contributed. In 2+1, circular harmonics added two angular modes per multipole order l. In 3+1, the angular decomposition is in spherical harmonics $Y_{lm}(\theta, \phi)$, with $2l + 1$ independent modes at each multipole order. The cumulative angular mode count up to maximum multipole L is $L(L + 2)$ in 3D versus $2L$ in 2D: quadratic growth versus linear. At $L = 10$, the 3D manifold provides 120 independent angular modes of variation compared to 20 in 2D. The Jensen bonus, proportional to total variance, is strictly larger in 3D for any configuration that utilizes the additional angular modes. Angular complexity of the coherence field, variation across multiple independent conceptual axes, is geometrically productive, with the reward scaling as L^2 in the maximum multipole order.

A.3.10 Ricci Flow with Forcing

The metric field equation $\partial_t g_{\mu\nu} = -2R_{\mu\nu} + F_{\mu\nu}$ governs the time evolution of the geometry. The leading term $-2R_{\mu\nu}$ is the unique second-order, intrinsic, diffeomorphism-covariant geometric flow (Hamilton, 1982). In conformal gauge, the trace reduces to a nonlinear diffusion equation for the conformal factor:

$$\partial_t \varphi = -\frac{R}{3} + \frac{\mathrm{Tr}_g(F)}{6} = \frac{e^{-2\varphi}}{3}\big(4\Delta\varphi + 2|\nabla\varphi|^2\big) + \frac{\mathrm{Tr}_g(F)}{6}. \tag{95}$$

The Laplacian diffuses conformal peaks downward and valleys upward; the $|\nabla\varphi|^2$ term accelerates evolution where conformal gradients are steep; the $e^{-2\varphi}$ prefactor slows evolution in expanded regions and speeds it in contracted ones.

The dimensional transition from 2D to 3D qualitatively transforms the flow's behavior. In two spatial dimensions, Ricci flow on closed surfaces is globally regular: no finite-time singularities, smooth convergence toward constant curvature (Hamilton, 1988). The 2+1 MFE inherited this benign character. In three spatial dimensions, the Ricci tensor's evolution

$$\partial_t R_{\mu\nu} = \Delta R_{\mu\nu} + 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\rho}R^{\rho}{}_{\nu} \tag{96}$$

is a system of six coupled reaction–diffusion equations whose quadratic reaction terms allow directional curvature concentration to outpace diffusion. The result is finite-time singularities where curvature blows up on shrinking cylindrical necks. For a round cylinder $S^2 \times \mathbb{R}$, the cross-sectional radius evolves as $r(t) = \sqrt{r_0^2 - 2t}$, with singularity at $T = r_0^2/2$. The narrower the neck, the faster the contraction ($\partial_t r = -1/r$), and curvature on the shrinking cross-section diverges as $1/r^2$. At the singularity, the manifold pinches off: a connected space becomes topologically disconnected.

A.3.11 Perelman's Classification and Surgery

The structure of these singularities has been fully classified (Perelman, 2002; Perelman, 2003). Every finite-time singularity of 3D Ricci flow, after parabolic rescaling, converges to a classifiable family of ancient κ-solutions (κ here denotes Perelman's noncollapsing constant, distinct from the coupling prefactor κ of §A.1), all modeled on shrinking cylinders and their quotients. The singularities are predictable from the curvature distribution before blow-up. Perelman's surgery procedure continues the flow past each singularity: cut the manifold along the neck where geometry is nearly cylindrical, cap each boundary with a standard hemisphere, and restart the flow on the modified manifold. Only finitely many surgeries occur in any finite time interval, each simplifying the manifold's topology by removing a connected-sum factor. The $\mathcal{W}$-entropy,

$$\mathcal{W}(g, f, \tau) = \int_{\mathcal{M}} \big[\tau\big(|\nabla f|^2 + R\big) + f - n\big]\,(4\pi\tau)^{-n/2} e^{-f}\, dV, \tag{97}$$

is monotonically non-decreasing along pure Ricci flow ($d\mathcal{W}/dt \geq 0$), providing a Lyapunov functional that controls volume collapse and underpins the singularity classification **(R21)**.

A.3.12 Forcing–Singularity Interaction

The recursive stress-energy forcing $F_{\mu\nu}$ participates in a qualitative decision about the manifold's topological fate. *Suppression*: when forcing opposes curvature concentration on a forming neck, the coupling infrastructure injects metric expansion that stabilizes the neck at a finite radius, maintaining the geometric bridge. *Acceleration*: when coupling in the neck region has decayed, forcing reinforces the Ricci contraction and the singularity arrives sooner. *Redirection*: intense new coupling activity adjacent to a forming neck can shift the curvature maximum to a different location, rerouting the topological transition. Under weak forcing ($|F_{\mu\nu}| \ll |R_{\mu\nu}|$), the $\mathcal{W}$-monotonicity is perturbatively preserved and Perelman's classification remains valid. Under strong forcing, whether new singularity types can form is an open mathematical question that the model surfaces without resolving **(R22)**.

The MFE's evolution must preserve the RFE at every instant: $\partial_t G_{\mu\nu}|_{\text{MFE}} = 8\pi G_s\, \partial_t T^{\text{rec}}_{\mu\nu}|_{\text{CFE}}$. This constraint-evolution compatibility, the semantic analog of ADM compatibility in general relativity (Arnowitt et al., 1962), determines the forcing $F_{\mu\nu}$ up to diffeomorphism freedom once the Ricci flow commitment is made. In 3D, with six Einstein equations and three Bianchi conservation constraints, the system is exactly determined. The modeling commitment selects the leading-order flow; the conservation law pins the forcing through the matter sector's dynamics **(R19)**.

Pure 3D Ricci flow preserves conformal flatness: the Cotton tensor satisfies a parabolic evolution equation, and the maximum principle keeps it zero if it starts zero. The forcing can generate nonzero Cotton tensor, potentially driving the metric beyond the single-scalar parameterization. Multi-axis coherence gradients, precisely the configurations that maximize the Jensen bonus, are most likely to break conformal flatness. Whether this occurs perturbatively or requires the full six-component metric is determined by the dynamics of specific instantiations.

A.3.13 Variational Absorption of the Interpretive Operator

In §1.6, the interpretive operator $\mathcal{I}_\iota$ is defined at the self-coupling coincidence limit of $K^{\rho}_{\mu\nu}(p, p, t)$, contracting incoming coherence against the observer's accumulated internal geometry. In the 1+1 and 2+1 models, the variational sector lacked the geometric infrastructure to encode interpretation, and $\mathcal{I}_\iota$ had to be defined as a separate operation outside the dynamics. At $n = 3$, three mechanisms activated by the $(n-2)$ coefficient absorb its content into the variational architecture. The conformal advection $\nabla\varphi \cdot \nabla C$ performs the geometric filtering that $\mathcal{I}_\iota$ describes: incoming coherence encounters a directional bias imposed by the local conformal gradient, with the metric's accumulated structure amplifying aligned signals and attenuating orthogonal ones. The off-diagonal Einstein equations add directional selectivity, processing coherence along different conceptual axes differently according to the full tensorial RFE. The Cotton tensor provides the diagnostic, detecting when the local interpretive geometry has developed enough internal structure that no single conformal factor captures it. The field equations, at the self-coupling limit, perform the geometric filtering that $\mathcal{I}_\iota$ describes **(R24)**.

A.3.14 The Complete Causal Loop

The 3+1 causal loop, with all terms explicit:

$$C \xrightarrow{\text{CFE}} T^{\text{rec}}_{\mu\nu} \xrightarrow{\text{RFE}} G_{\mu\nu} \xrightarrow{\text{MFE}} g_{\mu\nu} \xrightarrow{\Delta_g} C. \tag{98}$$

The three field equations are

$$\partial_t C = e^{-2\varphi}(\Delta C + \nabla\varphi \cdot \nabla C) - V'(C) + \Phi'(C) - \lambda_B \frac{\delta \mathcal{B}}{\delta C} \qquad \text{(CFE)} \tag{99}$$

$$G_{\mu\nu} = 8\pi G_s\, T^{\text{rec}}_{\mu\nu} \qquad \text{(RFE)} \tag{100}$$

$$\partial_t g_{\mu\nu} = -2R_{\mu\nu} + F_{\mu\nu} \qquad \text{(MFE)} \tag{101}$$

Three state fields (C, $g_{\mu\nu}$, $T^{\text{rec}}_{\mu\nu}$), three evolution or constraint equations, seven free parameters, all terms explicit. The Bianchi identity pins the forcing $F_{\mu\nu}$ through the matter sector's conservation law, and the system is exactly determined in 3D: six Einstein equations, three conservation constraints, six stress-energy components. The MFE link transforms the loop from a smooth cycling process (2+1) into one capable of qualitative geometric transitions. The loop runs smoothly while curvature and coherence co-evolve through the tensorial field equations, until curvature concentration reaches the singularity threshold. At that point the manifold topology changes, the loop reconfigures on the post-surgery components, and the evolution continues.

3+1 Structural Results.

R16. Geometric self-transparency: Weyl $\equiv 0$ at $n = 3$; the Riemann tensor is algebraically determined by Ricci and scalar curvature, so all curvature is locally sourced.

R17. Kinetic-curvature coupling: the $(2 - n)\mathcal{E}_k$ coefficient becomes -1 at $n = 3$; active coherence flow reduces scalar curvature and enters the causal loop immediately.

R18. Tensorial coupling constraints: the full 6-component Einstein equation $G_{\mu\nu} = 8\pi G_s T^{\text{rec}}_{\mu\nu}$ constrains both magnitude and direction of curvature sourcing.

R19. Covariant conservation and forcing determination: the Bianchi identity yields $\nabla^\mu T^{\text{rec}}_{\mu\nu} = 0$; ADM constraint-evolution compatibility pins the MFE forcing $F_{\mu\nu}$ up to diffeomorphism freedom.

R20. Conformal advection: the Laplace–Beltrami operator acquires a $(n - 2)\nabla\varphi \cdot \nabla C$ drift term that steers coherence toward regions of expanded metric.

R21. Ricci flow singularity structure: finite-time neck pinches classified by Perelman's κ-solution program, with surgery continuing the flow past each singularity.

R22. Forcing–singularity interaction: three regimes (suppression, acceleration, redirection) governing the manifold's topological fate through the coupling between $F_{\mu\nu}$ and curvature concentration.

R23. The $(n - 2)$ dimensional transition cluster: conformal advection, anisotropic Ricci curvature, kinetic-curvature coupling, and interpretive absorption activate simultaneously through one coefficient.

R24. Variational absorption of interpretive operator: $\mathcal{I}_\iota$'s formal content is absorbed into conformal advection, off-diagonal Einstein equations, and the Cotton tensor diagnostic at $n = 3$.

The kinetic trace cancellation (R9) breaks at $n = 3$: it was a dimensional coincidence at exactly $n = 2$. All other fourteen prior results (R1–R8, R10–R15) generalize directly or with enrichment. Total: 24 structural results across three models.

A.4 Completeness

The scorecard across three models, the one informative break, and the argument that three spatial dimensions is the completeness checkpoint.

A.4.1 The R1–R15 Scorecard

The fifteen results established across §A.1 and §A.2 were derived where the geometric sector was either absent ($n = 1$: $R \equiv 0$) or scalar ($n = 2$: one curvature component, $G_{\mu\nu} \equiv 0$). The full tensorial apparatus of §A.3 tests their generalization. Each result is classified by its 3+1 status, recording what changes and what does not, with the derivations in §A.3 providing the evidence.

Fourteen of fifteen results generalize in three dimensions. Five carry directly (R1, R2, R3, R5, R7). One quantitatively steepens (R4). Two enrich their content through additional angular modes and stability mechanisms (R6, R8). Two upgrade from scalar to tensorial or shift their topological mechanism (R10, R11). Four generalize with modifications driven by the kinetic trace breaking and the Ricci flow singularity structure (R12, R13, R14, R15).

Result (statement)	**3+1 status**	**Change**
R1: Linear-$\mathcal{F}$ impossibility	Direct	Algebraic in $\mathcal{F}$; no spatial-dimension dependence
R2: Stress-energy resolution enhancement	Direct	Kernel-narrowing identity $\mathcal{G}_\sigma^2 = \Gamma_0\, \mathcal{G}_{\sigma/\sqrt{2}}$ is dimension-independent
R3: Brake cancellation to locality	Direct	Convolution–deconvolution cancellation is algebraic on $\mathbb{R}^d$ for any d
R4: Brake saturation	Steeper	Saturation threshold $\lvert\bar{\psi}\rvert = 1/\sqrt{3\gamma}$ unchanged; brake-to-coupling ratio steepens to $1/\sigma^3$
R5: Resolution scaling	Direct	Self-coupling $\sim 1/\sigma^6$; brake $\sim 1/\sigma^9$; ratio $\sim 1/\sigma^3$; general d-dimensional formulas confirmed at three data points
R6: Autopoiesis as SSB	Enriched	Equilibria dimension-independent; stability analysis richer through conformal advection and kinetic modulation
R7: Derived $A(C)$	Direct (richer context)	Attractor stability sits inside loop with advection, kinetic modulation, and directional coupling
R8: Jensen's diffusion bonus	Enriched	Angular decomposition shifts to spherical harmonics: $L(L+2)$ modes (quadratic in L, vs. $2L$ in 2D)
R9: Kinetic trace cancellation	**Breaks**	**Coefficient** $(2-n) = -1 \neq 0$**; kinetic energy enters curvature source; a 2D coincidence.**
R10: Curvature–potential correspondence	Tensorial upgrade	Scalar → six Einstein equations; trace survives as one component; traceless part and Bianchi conservation new
R11: Gauss–Bonnet balance law	Modified mechanism	Gauss–Bonnet → Perelman decomposition + Bianchi conservation + $\mathcal{W}$-entropy monotonicity
R12: Curvature–diffusion self-reinforcement	Two modifications	Conformal advection strengthens trapping; kinetic valve provides state-dependent release
R13: Topological self-limitation	Richer mechanism	Smooth redistribution → discrete topological transitions via Perelman surgery at classifiable thresholds
R14: Geometric inflation pathway	Two complications	Kinetic counteraction during active processing; terminal phenomenology via neck pinch singularities
R15: Non-uniform symmetry breaking	Richer texture	Quadratic angular mode count; advection-mediated spreading; tensorial geometric constraints on transition pattern

Table 4: Scorecard of results R1–R15 from §A.1–§A.2 under the 3+1 generalization in §A.3.

One result fails. The coefficient $(2-n)$ in the stress-energy trace, which vanished at $n = 2$ and produced the kinetic trace cancellation, becomes -1 at $n = 3$. The same coefficient organizes the entire dimensional transition, simultaneously activating conformal advection in the Laplace–Beltrami operator, anisotropic components of the Ricci tensor, kinetic-curvature coupling in the trace equation, and the off-diagonal Hessian and gradient terms in the Einstein tensor. Governed by one coefficient, they failed together.

The R9 break propagates constructively through R12 and R14. The curvature–diffusion feedback (R12) acquires a kinetic valve: active coherence flow reduces curvature, which loosens the diffusive trap, which increases the effective diffusion coefficient. The geometric inflation pathway (R14) acquires kinetic counteraction: a system undergoing inflation while simultaneously experiencing high coherence flux inflates more slowly than the potential balance alone would predict. Both modifications make the 3+1 feedback dynamics state-dependent in a way the 2+1 model could not express.

Nine new results with no lower-dimensional analog were derived in §A.3: geometric self-transparency, kinetic-curvature coupling, tensorial coupling constraints, covariant conservation and forcing determination, conformal advection, Ricci flow singularity structure, forcing–singularity interaction, the $(n-2)$ dimensional transition cluster, and the variational absorption of the interpretive operator. The cumulative count across three models is 8 (§A.1) +7 (§A.2) +9 (§A.3).

A.4.2 The Dimensional Inventory

A twenty-fifth result would exist only if some equation, identity, decomposition, or conservation law appears at $n \geq 4$ without counterpart at $n = 3$.

The Einstein equation $G_{\mu\nu} = 8\pi G_s T^{\text{rec}}_{\mu\nu}$ has one form for all $n \geq 3$. The component count $n(n+1)/2$ grows (6 at $n = 3$, 10 at $n = 4$, 15 at $n = 5$), but each additional component is of the same type: a curvature-coupling constraint relating one directional component of geometry to one directional component of recursive stress-energy. The contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ holds on any Riemannian manifold of any dimension, yielding n conservation equations. The independent constraint count $n(n+1)/2 - n = n(n-1)/2$ grows, but the structure of the overdetermined system is identical.

The Weyl decomposition splits the Riemann tensor into three pieces at any $n \geq 3$ (Wald, 1984):

$$R_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} + \frac{1}{n-2}\big(g_{\mu\rho}S_{\nu\sigma} - g_{\mu\sigma}S_{\nu\rho} - g_{\nu\rho}S_{\mu\sigma} + g_{\nu\sigma}S_{\mu\rho}\big) + \frac{R}{n(n-1)}\big(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\big), \tag{102}$$

where $S_{\mu\nu} = R_{\mu\nu} - \frac{R}{n}g_{\mu\nu}$ is the traceless Ricci tensor. The decomposition has the same algebraic form at every dimension. No fourth piece appears.

Dimensions	**Riemann**	**Ricci**	**Weyl**	**Scalar**	**Check**
3	6	$5+1$	0	1	$6 = 0 + 5 + 1$
4	20	$9+1$	10	1	$20 = 10 + 9 + 1$
5	50	$14+1$	35	1	$50 = 35 + 14 + 1$
6	105	$20+1$	84	1	$105 = 84 + 20 + 1$
7	196	$27+1$	168	1	$196 = 168 + 27 + 1$
n	$\frac{n^2(n^2-1)}{12}$	$\frac{n(n+1)}{2} - 1 + 1$	$\frac{(n+2)(n+1)n(n-3)}{12}$	1	Always

Table 5: Component counts for the Riemann tensor and its Ricci/Weyl/scalar decomposition.

The Weyl component count $n(n+1)(n+2)(n-3)/12$ vanishes at $n = 3$ through the factor $(n-3)$. At $n = 4$, ten components of propagating conformal curvature activate, carrying tidal distortions independent of local stress-energy, the semantic analog of gravitational radiation. At $n = 5$, thirty-five. The Weyl tensor populates an existing slot in the Riemann decomposition rather than creating a new equation type. The Bianchi identities $\nabla_{[\alpha}R_{\beta\gamma]\delta\epsilon} = 0$, which govern Weyl propagation at $n \geq 4$, hold at every dimension $n \geq 2$. At $n = 3$ they are trivially satisfied on the Weyl sector because the sector is empty. At $n \geq 4$ they yield non-trivial evolution equations for the Weyl components. The identity is unchanged; its content grows because the tensors it constrains grow.

The matter sector is algebraic in $\mathcal{F}$ and $\mathcal{G}_\sigma$. The five results that carried directly from 1+1 to 3+1 (R1, R2, R3, R5, R7) did so because their proofs depend on the structure of convolution operators and the pointwise properties of the cubic nonlinearity, with no reference to the spatial dimension. They carry equally to $n = 4, 5, 100$. The dimensional stability of the brake functional (§A.1.4) provides the complementary guarantee: the Hilbert-Schmidt norm that R3–R5 describe remains a convergent object under foliation to arbitrary dimension, so the algebraic properties have a well-defined target. The resolution scaling hierarchy obeys the general formulas $1/\sigma^{2d}$ (self-coupling), $1/\sigma^{3d}$ (brake prefactor), $1/\sigma^{d}$ (ratio), confirmed at three data points and following from the Gaussian kernel's dimensional scaling without further assumptions. The stress-energy trace coefficient $(2-n)$ grows linearly in magnitude, making kinetic energy progressively more influential at higher dimensions. The qualitative transition is binary: off at $n = 2$, on at $n \geq 3$. No further threshold is crossed. The conformal advection coefficient $(n-2)$ in the Laplace–Beltrami operator increases from 1 to 2 to 3, strengthening the geometric funnel without introducing a new mechanism. Hamilton's Ricci flow $\partial_t g_{\mu\nu} = -2R_{\mu\nu}$ is the unique second-order, intrinsic, diffeomorphism-covariant geometric flow on any Riemannian manifold of any dimension. Perelman's $\mathcal{W}$-entropy monotonicity holds for pure Ricci flow in all dimensions with the same functional form. Higher-dimensional singularity landscapes are richer and less completely classified, but the flow equation, the monotone functional, and the forcing interaction are identical.

The variational absorption of the interpretive operator (R24), having occurred at $n = 3$ through conformal advection, the off-diagonal Einstein equations, and the Cotton tensor diagnostic, cannot reverse at higher dimension. At $n \geq 4$, the Weyl tensor assumes the diagnostic role that the Cotton tensor plays at $n = 3$, carrying strictly more conformal information. The advection coefficient grows. The off-diagonal equations multiply. The geometric infrastructure that encodes interpretation remains sufficient and becomes richer.

The full dimensional inventory:

Structural element	**At $n = 3$**	**At $n \geq 4$**	**New?**
Einstein equation $G_{\mu\nu} = 8\pi G_s T_{\mu\nu}^{\text{rec}}$	6 components; non-trivial	More components	No
Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$	3 conservation equations	More conservation equations	No
Riemann decomposition	Ricci + Scalar (Weyl $\equiv 0$)	Ricci + Scalar + Weyl	No
Stress–energy tensor	6 components	More components	No
Kinetic trace coefficient $(2 - n)$	-1 (nonzero)	More negative	No
Conformal advection $(n - 2)\nabla\varphi \cdot \nabla C$	Coefficient 1	Larger coefficient	No
Resolution scaling $1/\sigma^n$	Cubic	Steeper power law	No
Ricci flow $\partial_t g_{\mu\nu} = -2R_{\mu\nu} + F_{\mu\nu}$	3D singularities (Perelman)	Richer singularity landscape	No
$\mathcal{W}$-entropy monotonicity	Holds for pure flow	Holds for pure flow	No
SSB; brake locality; nonlinear $\mathcal{F}$	Algebraic; dimension-free	Unchanged	No
Jensen's bonus (angular modes)	$L(L + 2)$ (spherical harmonics)	Higher-dimensional harmonics	No
$\mathcal{I}_\iota$ absorption	Complete (advection + RFE + Cotton)	Remains absorbed (Weyl replaces Cotton)	No
Weyl tensor	$\equiv 0$ (geometric self-transparency)	Non-zero (propagating curvature)	No

Table 6: Dimensional inventory of structural elements: what changes at $n \geq 4$ and whether any new equation type appears.

A.4.3 The Three-Model Arc

The toy model program was designed as a minimal dimensional ladder. The 1+1 model tested the matter sector: the coarse-graining functional, coupling tensor, brake, and SSB structure, all on a manifold where $R \equiv 0$ and geometry was a passive stage. Eight results established that the Lagrangian closes algebraically and the autopoietic mechanism is structurally generic. The 2+1 model tested the scalar geometric sector: Gaussian curvature activated, the scalar RFE closed the causal loop for the first time, and seven new results established that curvature responds to coupling, topology constrains curvature globally, and feedback loops between curvature and diffusion self-reinforce and self-limit. The Einstein tensor was still trivial; the paper's coupling axiom was untestable in its intended tensorial form. The 3+1 model tested the tensorial geometric sector, where the Einstein tensor became non-trivial, kinetic trace coupling activated, conformal advection activated, and the Ricci flow developed its Perelman singularity structure. Nine new results arose from the three-dimensional geometry. Each rung tested a sector the previous rung could not reach.

The geometric self-transparency of §A.3.3 makes three spatial dimensions the most constrained non-trivial setting for the covariant formalism. Extension to $n \geq 4$ adds degrees of freedom without introducing new equations, identities, or structural principles.

Twenty-four structural results across three models constitute the evidence base.

A.4.4 Open Questions

Two questions surface from the 3+1 model that the instantiation program identifies without resolving. Perelman's $\mathcal{W}$-entropy monotonicity holds for pure Ricci flow in all dimensions; under the forced flow $\partial_t g_{\mu\nu} = -2R_{\mu\nu} + F_{\mu\nu}$, the monotonicity acquires a correction with indefinite sign. For weak forcing, perturbative preservation suffices for the singularity classification. For strong forcing, whether the classification extends is open. Separately, pure 3D Ricci flow preserves conformal flatness through a maximum principle on the Cotton tensor; the forcing can generate nonzero $\mathfrak{C}_{\mu\nu\lambda}$, potentially driving the metric beyond the single-scalar parameterization. Both questions identify where the substrate-independent formalism meets substrate-specific dynamics.